Modelling of multistage pendulums:

triple pendulum suspension for GEO 600

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Abstract:

Extensive dynamic six degree-of-freedom modelling of a multiple pendulum suspension has been developed for application to the design and analysis of the planned suspensions for the main mirrors in GEO 600, the German/UK gravitational wave detector. Two models were developed independently, and results from both were compared with experiment to verify their applicability. The models have been applied to investigate the optimisation of parameters for achieving the desired modal behaviour. In additions the levels of cross-coupling between degrees of freedom due to mechanical misalignments have been investigated and shown to be within acceptable limits.
1. Introduction

A very high degree of seismic isolation is required for the mirrors in interferometric gravitational wave detectors. Traditionally this is achieved by a combination of several stages of isolation, using for example acoustic isolation stacks and pendulums, or multiple pendulums [1, 2, 3, 4]. Pendulums are an excellent way to achieve good horizontal isolation. However their vertical isolation is much poorer due to the stiffness of the suspension wire. Increased vertical isolation can be achieved by incorporating soft vertical springs such as cantilever blades in the suspension, as is proposed for use in the VIRGO detector, in GEO 600 and in AIGA [1, 2, 5]. Vertical isolation is required since there will be at some level a cross-coupling of input vertical motion to resultant horizontal motion of the mirrors.

In GEO 600 we plan to achieve the required isolation by using a combination of an isolation stack consisting of three legs, each incorporating one active and one passive layer, and a triple pendulum incorporating two stages of cantilever blades as shown in figure (1). In this paper we present work carried out on modelling a triple pendulum to meet our design criteria. Full details of the final GEO 600 main suspension system are discussed in a companion paper [1]. Previous detailed work at Glasgow on modelling multiple pendulums was focused on a double pendulum suspension, as currently used in the Glasgow 10m prototype detector [6, 7].

The control philosophy used in GEO 600 requires that all the modes of the triple pendulum which are to be actively damped be resonant between 0.5 and 5 Hz [1]. Because the control is applied at the highest mass in the triple pendulum, each of these resonant modes has to couple strongly to this highest mass. To obtain an
acceptable design required investigation of various features such as number and spacing of wires, angling of the wires from the vertical, and adjustment of the points of attachment of the wires.

In GEO 600, a cross coupling factor of 0.1% from vertical to horizontal has been assumed. It is therefore important to establish that this factor is not exceeded in any practical triple pendulum design. The model has been used to investigate the sensitivity of this cross-coupling factor to small mechanical imperfections in the pendulum construction.

2. Conventions

The six degrees of freedom of a rigid body pendulum, illustrated in figure (2), are:

- Longitudinal motion—parallel to the x-axis and parallel to the input optical axis, making this the most important direction;
- Transverse motion—parallel to the y-axis, motion across the beam;
- Vertical motion—parallel to the z-axis, “up”;
- Tilt—rotation about y-axis;
- Rotation—rotation (yaw) about the z-axis; and
- Roll—rotation about the x-axis, which to first order does not disturb the optical beam.

The pendulum models to be described can handle a great variety of pendulum configurations. Most of the designs considered have certain aspects in common, such as being symmetric in the transverse direction. Every suspension has parameters similar to those shown in the single pendulum in figure (2), with additional lengths and separations for multiple stage pendulums:
\( l \) = the length of each wire,

\( s \) = the half separation of the wires in the x-direction,

\( t_1 \) = the half separation of the wires in the y-direction at the suspension point,

\( t_2 \) = the half separation of the wires in the y-direction at the mass \((t_2=t_1=t \text{ for vertical wires})\),

\( d \) = the distance the wires break-off above or below the plane through the centre of mass and

\( k \) = the spring constant of one wire.

3. Models

Two methods of modelling were begun, one writing down the differential equations of motion using Newton’s second law and the other by using the kinetic and potential energies in Lagrange’s equations. These methods proved complementary; some terms that were perhaps not obvious appear in the Lagrangian formulation while confirmation with the force method provided explanation and understanding of the results.

**a. Force Model**

The equations of motion for a triple pendulum suspension were written in a parameterised form as a MATLAB m-file. This code gives the appropriate state space coefficients for varying the wire lengths, sizes, materials, and attachment points and the sizes, shapes, and materials of the pendulum masses. It instantly generates the necessary matrices that interface with existing MATLAB control toolboxes. This model is the easiest and quickest way to parameterise the dynamic response of a compound pendulum.
This version of the model includes the forces due to tension in the wires and the spring restoring force of extension in the wires. It treats the wires as straight connections between the endpoints. These forces include all the dominant terms that affect the dynamics of the system for reasonable suspension design. Cantilevers are approximated by replacing the extension spring constant of the wire with the series sum of the spring constants of the wire and cantilever, which is dominated by the much softer cantilever.

b. Lagrangian Model

The second model uses the Maple symbolic math package to write out the kinetic and potential energy of the system. Using the Lagrangian formulation, the equations are expanded about equilibrium to form first order linear differential equations, which can be converted to a MATLAB state space formalism as needed.

The kinetic energy terms, treating the masses and cantilevers as rigid bodies, are easily expressed. The potential energy terms include the gravitational potential, the twisting of the wires, and the bending of the cantilevers, which prove to be straightforward; it also includes the bending and stretching of the wires, which are solved based on the shape of each wire. The wire shape is determined by solving the fourth order equation for a beam under tension. The detailed inclusion of these terms allows this code to include loss accurately and thus be used for thermal noise calculations. In particular, because of the way loss is included, generalised functions of loss, not restricted to constant viscous or structural loss, can be included.

The cantilever positions are included as additional state variables and result in another set of equations in the model.
Because the Lagrangian code generates equations of motion from scratch, there is a tremendous amount of flexibility as to what can be modelled. Perturbations in individual wire size, length, or point of attachment are simple to include.

c. Applications of the models

There were clear advantages to having both of these models available for design and analysis. The MATLAB model, which took advantage of presumed symmetries, was useful for selecting parameters to optimise the design, and for readily modelling servo systems using existing MATLAB control packages suitably adapted. The simplifying assumptions meant that candidate designs could be quickly assessed. In contrast the Lagrangian model was more comprehensive. For example when the wires were being modelled the program fully incorporated their bending stiffness. Further there were no initial assumptions made about symmetries in the system. Thus analytical computations took much longer, but could be expanded to investigate cross-coupling due to misalignments in construction, and to examine thermal noise performance. The model could also be extended for example to investigate non-symmetric suspensions.

d. Evaluation

The two models were compared in great detail throughout their mutual development. When possible, the explicit symbolic equations were cross checked. Both were also checked extensively against prior double pendulum work [7] so that each detail and enhancement was well understood.
4. Single Pendulum Modelling

a. Single Pendulum with vertical wires

The simplest example is for a single pendulum suspended from four vertical wires breaking off from the mass in a plane level with the centre of mass. In this case the equations for the vertical, longitudinal, tilt and rotation modes of a single pendulum are uncoupled to first order and the frequencies can be calculated from:

$$\omega_{\text{vertical}}^2 = \frac{4k}{m} \quad \text{Equation (1)}$$

$$\omega_{\text{longitudinal}}^2 = \frac{g}{l} \quad \text{Equation (2)}$$

$$\omega_{\text{tilt}}^2 = \frac{4ks^2}{I_Y} \quad \text{Equation (3)}$$

$$\omega_{\text{rotational}}^2 = \frac{mg(s^2+r^2)}{lI_Z} \quad \text{Equation (4)}$$

where $\omega = 2\pi f$, $f$ = resonant frequency, $I_Y$ = moment of inertia about the y-axis, $I_Z$ = moment of inertia about the z-axis, $m$ = mass and $g$ = the acceleration due to gravity.

b. Single Pendulum: Effect of Angling the Suspension Wires

Careful consideration of the force vectors explains the additional terms that appear in the dynamic equations when the suspension wires are angled. In particular, consider the equation for vertical motion for a single pendulum of mass $m$ suspended from four wires of length $l$ with spring constant $k$ and at an angle of $\Omega$ with the vertical as in figure (2).
\[ m \frac{d^2 z}{dt^2} = -4kz \cos^2 \Omega - \frac{z \sin^2 \Omega}{l} \frac{mg}{\cos \Omega} \quad \text{Equation (5)} \]

Effectively there are two restoring terms. The first is the force due to the extension of the wire. The additional term comes from a change in the angle, \( \Omega \), of the wires when the pendulum moves in the vertical direction. The terms for the other degrees of freedom follow in a similar fashion, giving the following resonant frequencies:

\[ \omega^2_{\text{vertical}} = \frac{4k \cos^2 \Omega}{m} + \frac{g \sin^2 \Omega}{l \cos \Omega} \quad \text{Equation (6)} \]

\[ \omega^2_{\text{longitudinal}} = \frac{g}{l \cos \Omega} \quad \text{Equation (7)} \]

\[ \omega^2_{\text{tilt}} = \frac{4ks^2 \cos^2 \Omega}{I_Y} + \frac{mg s^2 \sin^2 \Omega}{I_Y l \cos \Omega} \quad \text{Equation (8)} \]

\[ \omega^2_{\text{rotational}} = \frac{mg(s^2 \cos^2 \Omega + t_1 t_2)}{II_z \cos \Omega} + \frac{4ks^2 (t_2 - t_1)^2}{l^2 I_z} \quad \text{Equation (9)} \]

In comparing equations 1-4 with equations 6-9 it can be seen that the longitudinal equation is the same as before except for the projection of the tension term that has an angular dependence on \( \Omega \). It was initially less obvious that there should be additional terms in the vertical, rotation, and tilt equations which depend upon \( g/l \). It can now be seen that these terms come from changes in the angles of the wires when the pendulum moves in either vertical or rotational directions.

For vertical wires, the behaviour of the sideways and roll modes is very similar to the longitudinal and tilt modes. Angling the wires causes these pairs of modes to be coupled together. Similarly, if the wires connect to the mass out of the plane of the centre of mass, motion is coupled between translation and rotation. For example, for the longitudinal and tilt directions the resonant frequencies are the eigenvalues of the dynamics matrix, \([A]\), where \( x \) represents longitudinal motion and \( \theta \) represents tilt about the transverse axis, such that:
\[
\begin{bmatrix}
\frac{d^2 x}{dt^2} \\
\frac{d\theta}{dt}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
x \\
\theta
\end{bmatrix}
\]

Equation (10)

where

\[
A_{11} = \frac{-g}{l \cos \Omega}, 
A_{12} = \frac{gd}{l \cos \Omega}, 
A_{21} = \frac{mgd}{l \cos \Omega}, 
\text{and}
\]

\[
A_{22} = \frac{-4k s^2 \cos^2 \Omega}{l_y} - \frac{mgs^2 \sin^2 \Omega}{ll_y \cos \Omega} - \frac{mgd}{l_y} - \frac{mgd^2}{ll_y \cos \Omega}.
\]

Equations (11-14)

c. Experimental Results

To test the model, a single pendulum was set up on a four-wire suspension. The resonant frequencies were obtained by exciting the pendulum and measuring the Fourier response from an accelerometer attached to the mass. The experimental results were obtained for four different examples. The first case investigated had four vertical wires breaking off level with the centre of mass. Secondly the wires were angled in the y-direction for two different angles in order to confirm the new equations. Finally, further coupling was introduced by having the wires break off above the plane through the centre of mass. For all the cases the mass, \( m \), was 20 kg. The various cases and parameters for each are summarised in table I.

The first theoretical predictions gave a value too high compared to the measured value for the frequencies where the spring constant dominated. Most of the physical parameters are known with confidence; the single unverified input is the spring constant of the wire, given by

\[
k = \frac{E \pi r^2}{l},
\]

Equation (15)

where \( r \) is the radius of the wire, \( l \) is the length of the wire (both of which are simple to measure), and \( E \) is the Young’s Modulus. For these experiments, the suspension wire is made of stainless steel 302, of diameter from 280 to 350 \( \mu \)m. In the first set of
predictions a reference value of Young’s Modulus of $2.0 \times 10^{11}$ Pa [8] was used. To account for the discrepancies, an attempt was then made to fit the theory using the measured vertical frequencies, which depend most dominantly on the spring constant, to calculate a new value for $E$. The new modulus was then used to predict the other frequencies. However this over corrects, giving theoretical frequencies in tilt, rotation and roll that were too low compared to measured values. It was postulated that some of the vertical frequencies might be affected by coupling to the support structure which was not completely rigid; this idea was supported by some simple adjustments to the experimental setup. Thus it was decided to independently measure the modulus of the wire by measuring the vertical and rotational frequencies of a different pendulum, as long as practical (8 m) and mounted directly to the building to minimise the effects of the support structure. These measured frequencies each gave a value for $E$ of $\sim 1.7 \times 10^{11}$ Pa.

Using this value the theoretical predictions agree with the measured results within the experimental error of 0.1 Hz, as shown in table I, except for the vertical frequencies for our first three cases. In these cases, it is believed that the support structure still couples with the pendulum to give a lower frequency, as varying the bracing on the test structure caused the measured vertical frequency to vary significantly. The last case was measured on a different support structure which was noticeably stiffer and consequently gives better agreement.

Further experimental verification was carried out for multiple stage pendulums. This served to confirm our modelling work for representative double and triple pendulum suspensions, with and without cantilever elements included.
5. Extension to Double and Triple Pendulums

a. Applications of the models

The modelling for the multiple stage pendulums follows from the single pendulum modelling. In principle, the stages are cascaded together. The models are then used to assist in the design of the triple pendulum suspension in figure (1).

The control to be used relies on all frequencies that are to be damped to be between 0.5 and 5 Hz. (Details of the design philosophy and particulars for GEO 600 are found in the companion paper [1].) However, initial designs using vertical wires had a rotational (yaw) frequency that was too high. Some design parameters are constrained (for example, the size of the mirrors for optical and manufacturing reasons). Free parameters include the separation of the suspension wires in the x-direction and their angle from vertical in the y-direction. To investigate the dependence of the mode frequencies on these free parameters, it is a simple matter to vary these inputs to the force model.

Figure (3) shows how the tilt frequency is affected by the separation of the wires in the x-direction. This example shows a four wire suspension where the wires are attached above the centre of mass \( (d \neq 0) \) and the same setup incorporating vertical springs of low spring constant (cantilevers). Specifically, this case models a test mass \( 0.09 \text{ m} \) in radius, \( 0.10 \text{ m} \) in length, mass \( \sim 6 \text{ kg} \). The steel wires, of radius 198 \( \mu\text{m} \), are at \( t_1=t_2=0.0995 \text{ m} \) (which is the mass radius plus a small break off bar), attached 1 mm above the plane of the centre of mass, and are 0.18 m in length. The cantilevers had a spring constant of \( 1.3 \times 10^3 \text{ N/m} \). The figure shows the two effects that contribute to the tilt frequency calculated by equations (10-14). For large separations, the restoring force is due to the stretching of wire, and the frequency varies linearly with the
separation, \( s \). For small separations, the restoring force is dominated by the tension in the wires acting through the lever arm \( d \), the distance above the centre of mass, and the frequency is constant with \( s \).

In a similar fashion, figure (4) shows how the rotational frequency is affected by the upper separation of the wires in the y-direction, for \( t_1 < t_2 \), \( t_1 = t_2 \) and \( t_1 > t_2 \) where \( t_2 = 9.5 \text{ cm} \), again for a four wire suspension and for four wires with cantilevers. This case is for the same parameters as above with a separation of the wires in the \( x \)-direction of \( s = 3 \text{ cm} \). These parameters are comparable with the intermediate stage of a GEO 600 suspension. Here we see that, from equation (19), as \( t_1 \) decreases, the stiffness contributed by the stretching of the wires increases, raising the resonant frequency. When the cantilevers are used, the relative stiffness in extension, \( k \), is small enough that the rotation frequency drops with the increasing angle from vertical, \( \Omega \).

This kind of information can be used to facilitate the selection of these parameters for a pendulum suspension to get the desired resonant frequency behaviour and overall transfer function. An example of the horizontal transfer function, between the top point of the suspension and 1 mm above the centre of mass of the test mass, for a prototype GEO 600 triple pendulum is shown in figure (5). The three longitudinal and one tilt mode can be seen and the fall off with frequency above these modes is \( 1/f^6 \) as expected. In principle all of the longitudinal and tilt modes can be seen in the transfer function by looking further away from the centre of mass of the test mass. Further work on investigating the overall isolation performance of GEO 600 using the appropriate transfer functions generated by these models can be found in the companion paper [1].
b. Comparison with Experiment

The final experiment was for a triple pendulum with two sets of cantilevers, one stage supporting the top of the pendulum and one between the upmost and intermediate pendulum stages. The parameters used were determined through the kinds of tools described above, and match those for the design of the GEO 600 suspension system. For the prototype GEO 600 suspension, the specific results are shown below in table II. The agreement for the most part is excellent. Due to the experimental setup, the sideways modes were difficult to detect. Additionally, the cluster of modes near 0.5 Hz made it difficult to explicitly resolve all the modes.

6. Cross-coupling

Cross couplings in a suspension due to mechanical imperfections may restrict the actual achieved performance. For example, in a pendulum supported on four vertical wires attached above the centre of mass, if one wire has a different diameter and thus a different spring constant, input vertical motion will cause the mirror to tilt, and since tilt motion couples to longitudinal motion, horizontal motion will be observed for pure vertical input. This is a difficult coupling to quantify precisely in experiment, and in design is often assumed to take a constant value between 0.1-1%. This ignores the fact that the mechanical coupling, which depends on the dynamics, will certainly have some frequency dependence. For GEO 600, the assumed level of cross coupling used in design is 0.1%. This coupling results largely from the folding of the optical path in the vertical direction in the arms of the interferometer, which gives a value on the order of 0.04%. The conservative 0.1% value allows some small amount of extra coupling due to mechanical factors.
The Lagrangian model can be used to calculate the transfer function between vertical input and longitudinal output to predict the level of cross coupling, the ratio of horizontal output to vertical output for the same vertical input. Figure (6) shows one example, using parameters similar to those anticipated for the final stage of the GEO 600 main suspension; these include a 9 cm radius, 10 cm length fused silica mass (5.6 kg), and 4 fused silica wires of radius 125 µm, with $s=5$ mm, $t_1=t_2=9.65$ cm, and $d=1$ mm. As this is intended to be an entirely fused silica stage using silicate bonding of the fibres [9], when in situ at the GEO 600 site it will be hardest to adjust the position of the fibres of this stage, as opposed to metal wires which could be clamped and reclamped to their proper position. The figure shows the cross coupling at 50 Hz as we vary the attachment point of one wire and the spring constant, $k$, of that same wire, sensed 1 mm below the centre of mass to deliberately introduce additional tilt motion. Varying the spring constant corresponds to variations in fibre diameter. From our analyses we found that for frequencies well above the pendulum resonances of the system, and below any internal resonances, the vertical and horizontal motions are being attenuated as the same function of frequency, giving effectively constant cross-coupling. Thus our assumption of a fixed cross-coupling factor in the region of 50 Hz is valid. The results suggest that the assumed GEO 600 figure of 0.1% coupling, which includes the intrinsic coupling due to the details of the optical path and the coupling due to mechanical imperfections as shown in the figure, is appropriate, even for relatively large errors in wire parameters.

We note that at frequencies around the pendulum resonances the cross-coupling may be significantly larger than the level at higher frequencies [10]. This could be an important consideration for future detectors operating to lower frequencies.
7. Further design work

The modelling described here has been used to design the main suspension for the GEO 600 detector. The state space matrices generated were used to predict and confirm the behaviour of the local control servos used in that suspension.

Once the dynamics of a pendulum suspension have been calculated, it is straightforward to calculate any transfer function of interest, such as from either ground input or control actuators. By including loss in the system, as an imaginary part of a material’s Young’s modulus, the same dynamic equations can solve the impedance of the system, which give the resultant thermal noise from the Fluctuation-Dissipation theorem [11]. The conclusions involving the thermal noise that result from this modelling will be discussed in a future paper.

The flexibility of the modelling code allows investigation of dramatically different types of suspensions. Rather than wire based symmetric suspensions, it is straightforward to replace wires with simple flexures, or to test five wire or other reduced degree of freedom geometries.

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References


Table (I): Mode Frequencies of Single Pendulum

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<th>Case</th>
<th>All in (Hz)</th>
<th>Tilt</th>
<th>Longitudinal</th>
<th>Roll</th>
<th>Sideways</th>
<th>Rotation</th>
<th>Vertical</th>
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<td>1.78</td>
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<tr>
<td><strong>Experimental</strong></td>
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<td>1.0</td>
<td>1.9</td>
<td>11.9</td>
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<td>1.01</td>
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<td>1.5</td>
<td>11.4</td>
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Case 1: \( l=0.26 \text{m}, \ s=0.025 \text{m}, \ t_1=0.047 \text{m}, \ t_2=0.133 \text{m}, \ d=0 \)

Case 2: \( l=0.27 \text{m}, \ s=0.04 \text{m}, \ t_1=0.133 \text{m}, \ t_2=0.133 \text{m}, \ d=0 \)

Case 3: \( l=0.40 \text{m}, \ s=0.025 \text{m}, \ t_1=0, \ t_2=0.133 \text{m}, \ d=0 \)

Case 4: \( l=0.36 \text{m}, \ s=0.025 \text{m}, \ t_1=0, \ t_2=0.133 \text{m}, \ d=0.032 \text{m} \)
Table (II): Mode Frequencies of Prototype Triple Pendulum

<table>
<thead>
<tr>
<th>All in (Hz)</th>
<th>Theoretical</th>
<th>Experimental</th>
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<td></td>
<td>1.34, 0.5, 0.6</td>
<td>1.4, 0.6</td>
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<td>1.4, 1.0, 0.6</td>
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<td></td>
<td>1.4, 0.9, 0.6</td>
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</tr>
<tr>
<td>Rotational</td>
<td>3.1, 1.6, 0.4</td>
<td>3.1, 1.6</td>
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<tr>
<td>Vertical</td>
<td>36, 3.8, 1.0</td>
<td>37, 3.7, 1.0</td>
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Figure (1): The triple pendulum suspension in GEO 600

Figure (2): Schematic of a single pendulum

Figure (3): Tilt frequency as a function of the separation of the wires in the x-direction

Figure (4): Rotational frequency as a function of the upper separation of the wires in the y-direction

Figure (5): Horizontal transfer function of the triple pendulum from the top point of suspension to 1 mm above the centre of mass of the test mass

Figure (6): Cross-coupling at 50 Hz between horizontal output and vertical output, due to vertical input, observed 1 mm below the centre of mass
Figure (1): Husman, Rev. Sci. Instrum.
Figure (2): Husman, *Rev. Sci. Instrum.*
Figure (3): Husman, *Rev. Sci. Instrum.*
Figure (4): Husman, *Rev. Sci. Instrum.*
Figure (5): Husman, *Rev. Sci. Instrum.*
Figure (6): Husman, *Rev. Sci. Instrum.*