Generation of Hyperentangled Photon Pairs

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We experimentally demonstrate the first quantum system entangled in every degree of freedom (hyperentangled). Using pairs of photons produced in spontaneous parametric down-conversion, we verify entanglement by observing a Bell-type inequality violation in each degree of freedom: polarization, spatial mode, and time energy. We also produce and characterize maximally hyperentangled states and novel states simultaneously exhibiting both quantum and classical correlations. Finally, we report the tomography of a $2 \times 2 \times 3 \times 3$ system (36-dimensional Hilbert space), which we believe is the first reported photonic entangled system of this size to be so characterized.

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Entanglement, the quintessential quantum mechanical correlations that can exist between quantum systems, plays a critical role in many important applications in quantum information processing, including the revolutionary one-way quantum computer [1], quantum cryptography [2], dense coding [3], and teleportation [4]. As a result, the ability to create, control, and manipulate entanglement has been a defining experimental goal in recent years. Higher-order entanglement has been realized in multiparticle [5] and multidimensional [6–9] systems. Furthermore, two-component quantum systems can be entangled in every degree of freedom (DOF), or hyperentangled [10]. These systems enable the implementation of 100%-efficient complete Bell-state analysis with only linear elements [11] and techniques for state purification [12]. In addition, hyperentanglement can also be interpreted as entanglement between two higher-dimensional quantum systems, offering significant advantages in quantum communication protocols (e.g., secure supersdense coding [13] and cryptography [14]).

Photon pairs produced via the nonlinear optical process of spontaneous parametric down-conversion have many accessible DOF that can be exploited for the production of entanglement. This was first demonstrated using polarization [15,16], but the list expanded rapidly to include momentum (linear [17], orbital [6], and transverse [18] spatial modes), energy time [19] and time bin [20], simultaneous polarization and energy time [21], and recently, simultaneous polarization and 2-level linear momentum [22]. In this work, we produce and characterize pairs of single photons simultaneously entangled in every DOF—polarization, spatial mode, and energy time. As observed previously [6], photon pairs from a single nonlinear crystal are entangled in orbital angular momentum (OAM). Moreover, polarization entangled states can be created by coherently pumping two contiguous thin crystals [23], provided the spatial modes emitted from each crystal are indistinguishable. Finally, the pump distributes energy to the daughter photons in many ways, entangling each pair in energy; equivalently, each pair is coherently emitted over a range of times (within the coherence of the continuous wave pump). We show our two-crystal source can generate a $(2 \times 2 \times 3 \times 3 \times 2 \times 2)$-dimensional hyperentangled state [10], approximately

$$\begin{aligned}
&\left(\begin{array}{c}
|HH\rangle + |VV\rangle \\
(|rl\rangle + \alpha |gg\rangle + |lr\rangle) \otimes (|ss\rangle + |ff\rangle).
\end{array}\right)
\end{aligned}$$

Here $H$ ($V$) represents the horizontal (vertical) photon polarization; $|0\rangle$, $|g\rangle$, and $|r\rangle$ represent the paraxial spatial modes (Laguerre-Gauss) carrying $-\hat{h}$, 0, and $+\hat{h}$ OAM, respectively [24]; $\alpha$ describes the OAM spatial-mode balance prescribed by the source [25] and selected via the mode-matching conditions; and $|s\rangle$ and $|f\rangle$, respectively, represent the relative early and late emission times of a pair of energy anticorrelated photons [19].

The most common maximally entangled states are the 2-qubit Bell states: $\Phi^\pm = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $\Psi^\pm = (|01\rangle \pm |10\rangle)/\sqrt{2}$, in the logical basis $|0\rangle$ and $|1\rangle$. By collecting only the $\pm \hat{h}$ OAM state of the spatial subspace, the state (1) becomes a tensor product of three Bell states $\Phi_{\text{poln}} \otimes \Phi_{\text{spa}} \otimes \Phi_{\text{CE}}$. As a preliminary test of the hyperentanglement, we characterized the polarization and spatial-submode spaces by measuring the entanglement (characterized by tangle $T$ [26]), the mixture (characterized by linear entropy $S_L(\rho) = 1 - \frac{1}{4}[1 - \text{Tr}(\rho^2)]$ [27]), and the fidelity $F(\rho, \rho_i) = \text{Tr}(\sqrt{\sqrt{\rho} \rho_i \sqrt{\rho}})^2$ of the measured state $\rho$ with the target state $\rho_i = |\psi_i\rangle \langle \psi_i|$. We consistently measured high-quality states with tangles, linear entropies, and fidelities with $\Phi^+$ of $T = 0.99(1)$, $S_L = 0.01(1)$, and $F = 0.99(1)$ for polarization; and $T = 0.96(1)$, $S_L = 0.03(1)$, and $F = 0.95(1)$ for spatial mode, significantly higher than earlier results [18].

The experiment is illustrated in Fig. 1. A 120 mW 351 nm Ar$^+$ laser pumps two contiguous $\beta$-barium borate (BBO) nonlinear crystals with optic axes aligned in perpendicular planes [23]. Each 0.6 mm thick crystal is phase matched to produce type-I degenerate photons at 702 nm.
into a cone of 3.0° half-opening angle. The first (second) crystal produces pairs of horizontally (vertically) polarized photons, and these two possible down-conversion processes are coherent, provided the spatial modes emitted from each crystal are indistinguishable. With the pump focused to a waist at the crystals, this constraint can be satisfied by using thin crystals and “large” beam waists (large relative to the mismatch in the overlap of the down-conversion cones from each crystal [23]). However, the OAM entanglement is maximized by balancing the relative populations of the low-valued OAM eigenstates [25], which requires smaller beam waists to image a large area of the down-conversion cones. Here we compromise by employing an intermediate waist size (≈90 μm) at the crystal.

Mode-matching lenses are then used to optimize the coupling of the rapidly diverging down-conversion modes into single-mode collection fibers.

The measurement process consists of three stages of local state projection, one for each DOF. At each stage, the target state is transformed into a state that can be discriminated from the other states with high accuracy. Specifically, computer-generated phase holograms [28] transform the target spatial mode into the pure Gaussian (or 0-OAM) mode, which is then filtered by the single-mode fiber [6] [Fig. 1(b)]. After a polarization controller to compensate for the fiber birefringence, wave plates transform the target polarization state into horizontal, which is filtered by a polarizer [Fig. 1(d)]. The analysis of the energy-time state is realized by a Franson-type [19] polarization interferometer without detection postselection [21]. The matched unbalanced interferometers give each photon a fast |f⟩ and slow |s⟩ route to its detector. Our interferometers consisted of L ≈ 11 mm quartz birefringent elements, which longitudinally separated the horizontal and vertical polarization components by Δn_{\text{quartz}}L ≈ 100 μm, more than the single-photon coherence length (λ^2/Δλ ≈ 50 μm with Δλ = 10 nm from the interference filters) but much less than the pump-photon coherence length (∼10 cm). We rely on the photons’ polarization entanglement |HH⟩ + |VV⟩ to thus project onto a two-time state (H_{s}H_{s} + e^{i(δ_1 + δ_2)}|V_{f}V_{f}⟩), where δ_1 and δ_2 are controlled by birefringent elements (liquid crystals and quarter-wave plates) in the path of each photon [21].

Finally, by analyzing the polarization in the ±45° basis, we erase the distinguishing polarization labels and can directly measure the coherence between the |ss⟩ and |ff⟩ terms, arising from the energy-time entanglement.

To verify quantum mechanical correlations, we tested every DOF against a Clauser-Horne-Shimony-Holt (CHSH) Bell inequality [29]. The CHSH inequality places constraints (S ≤ 2) on the value of the Bell parameter S, a combination of four two-particle correlation probabilities using two possible analysis settings for each photon. If S > 2, no separable quantum system (or local hidden variable theory) can explain the correlations; in this sense, a Bell inequality acts as an “entanglement witness” [30]. To measure the strongest violation for the polarization and spatial-mode DOFs, we determined the optimal measurement settings by first tomographically reconstructing the 2-qubit subspace of interest; we employ a maximum likelihood technique to identify the density matrix most consistent with the data [27].

Table I shows the Bell parameters measured for the polarization, spatial-mode, and energy-time subspaces, with various projections in the complementary DOF. We see that for every subspace, the Bell parameter exceeded the classical limit of S = 2 by more than 20 standard deviations (σ), verifying the hyperentanglement. For both the polarization and spatial-mode measurements, we traced over the energy-time DOF by not projecting in this subspace. We measured the polarization correlations while projecting the spatial modes into the orthogonal basis states (|l⟩, |g⟩, and |r⟩), as well as the superpositions |h⟩ = (|l⟩ + |r⟩)/√2 and |v⟩ = (|l⟩ − |r⟩)/√2. The measured Bell parameters agreed (within ∼2σ) with predictions from tomographic reconstruction and violated the inequality by more than 30σ. In the spatial-mode DOF, the corre-

**FIG. 1.** Experimental setup for the creation and analysis of hyperentangled photons. (a) The photons, produced using adjacent nonlinear crystals (BBO), pass through a state filtration process for each DOF before coincidence detection. The measurement inlets show the filtration processes as a transformation of the target state (dashed box) and a filtering step to discard the other components of the state (dotted box). (b) Spatial filtration (spa): hologram (holo) and single-mode fiber (smf). (c) Energy-time transformation (e-t): thick quartz decoherer (dec) and liquid crystal (LC). (d) Polarization filtration (poln): quarter-wave plate (qwp), half-wave plate (hwp), and polarizer (pol).
lations for the state $\Phi^+_{\text{spa}}$ were close to maximal ($S = 2\sqrt{2} = 2.83$), also in agreement with predictions from the measured state density matrix. In addition, we tested Bell inequalities for nonmaximally entangled states in the OAM subspace: $\alpha|gg\rangle + |rl\rangle$ and $\alpha|gg\rangle + |lr\rangle$; the measured Bell parameters in this case were slightly smaller (5%, maximum) than predictions from tomographic reconstruction [31], yet still 20% above the classical limit. Finally, our measured Bell violation for the energy-time DOF using particular phase settings is in good agreement with the prediction ($S = 2\sqrt{2}$) from the measured 2-photon interference visibility $V = 0.985(2)$.

The polarization and spatial-mode state was fully characterized via tomography [27]. We performed the 1296 linearly independent state projections required for a full reconstruction in the $(2 \otimes 3) \otimes (2 \otimes 3)$ Hilbert space consisting of two polarization and three OAM modes for each photon. The measured state (Fig. 2) overlaps the anticipated state [polarization and spatial DOFs of Eq. (1)] with a fidelity of 0.69(1) for $\alpha = 1.88e^{0.16i\pi}$ (numerically fitted), and $S_L = 0.46(1)$, suggesting the difference arises mostly from mixture. Treating the photon pairs as a six-level two-particle system, we can quantify the entanglement using the negativity $N$ [32]. In this $6 \otimes 6$ Hilbert space, $N$ ranges from 0 (for separable states) to 5 (for maximally entangled states), and the fitted state above has $N = 4.44$. Our measured partially mixed state has $N = 2.96(4)$, indicating strong entanglement. The spatial mode alone has $N = 1.14(2)$, greater than the maximum ($N = 1$) of any two-qubit system. Thus, our large state possesses 2-qubit and 2-qutrit entanglement.

We also selected a state [neglecting the $|gg\rangle$ component, Fig. 3(a)] maximally entangled in both polarization and spatial mode that had $F = 0.974(1)$ with the target $\Phi^+_{\text{poln}} \otimes 260501-3$.

FIG. 2 (color online). Measured density matrix ($\rho$) and close pure state $|\Psi_p\rangle = \Phi^+_{\text{poln}} \otimes (|rl\rangle + \alpha|gg\rangle + |lr\rangle)$ with $\alpha = 1.88e^{0.16i\pi}$ of a $(2 \times 2 \times 3 \times 3)$-dimensional state of 2-photon polarization and spatial mode [35].

FIG. 3 (color online). Measured density matrices (real parts) of $(2 \times 2 \times 2 \times 2)$-dimensional states of 2-photon polarization and (+ 1, −1)-qubit OAM [35]. For each state, we list the target state $\rho_t$, the fidelity $F(\rho, \rho_t)$ of the measured state $\rho$ with the target $\rho_t$, their negativities and linear entropies, and the tangle and linear entropy for each subspace. The negativity for two-qubit states is the square root of the tangle. The magnitudes of all imaginary elements, not shown, are less than 0.03.
\[ \Phi_{\text{spa}}. \] By tracing over polarization (spatial mode), we look at the measured state in the spatial-mode (polarization) subspaces. The reduced states in both DOFs are pure \( (S_k < 0.04) \) and highly entangled \( (T > 0.94). \)

With this precise source of hyperentanglement, we have the flexibility to prepare nearly arbitrary polarization states \([33]\), and to select arbitrary spatial-mode encodings. For example, we also generated a different maximally entangled state: \( \Phi^+_{\text{pol}} \otimes \Phi^+_{\text{spa}} \) [Fig. 3(b)]. By coupling to and tracing over the energy-time DOF using quartz decoherers \([33]\), we can add mixture to the polarization subspace, allowing us to prepare a previously unrealized state that simultaneously displays classical correlations in polarization and maximal quantum correlations between spatial modes [Fig. 3(c)]: \( \rho = \frac{1}{2}([|HH\rangle\langle HH| + |VV\rangle\langle VV|]) \otimes |\Phi_{\text{spa}}^+\rangle \otimes |\Phi_{\text{pol}}^+\rangle \). We were also able to accurately prepare the state \( \rho_T = \frac{1}{T} |\Phi_{\text{pol}}^+\rangle \otimes |\Phi_{\text{spa}}^+\rangle \), with no polarization correlations at all (i.e., completely mixed or unpolarized), while still maintaining near maximal entanglement in the spatial DOF [Fig. 3(d)].

We report the first realization of hyperentanglement of a pair of single photons. The entanglement in each DOF is demonstrated by violations of CHSH-Bell inequalities of greater than \( 20\sigma \). Also, using tomography we fully characterize a \( 2 \otimes 2 \otimes 3 \otimes 3 \) state, the largest quantum system to date. In restricted \( (2 \times 2 \times 2 \times 2) \)-dimensional subspace, we prepare a range of target states with unprecedented fidelities for quantum systems of this size, including novel states with a controllable degree of correlation in the polarization subspace. These hyperentangled states enable 100%-efficient Bell-state analysis \([11]\), which is important for a variety of quantum information protocols \([3,13]\). Because the spatial-mode and energy-time DOFs are infinite in size, we envision examining even larger subspaces, encoding higher-dimensional qudits \([7,8]\).

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[26] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998); \( T(\rho) = \frac{1}{\sqrt{2}} \sum_{i=0}^{3} \rho_{i, i} \) where \( \rho_{i, i} = \sum_{j=0}^{3} \rho_{i, j} \) are the square roots of the eigenvalues of \( \rho = \rho_{i, i} \).
[28] Binary plane-wave phase gratings [24] (<40% diffraction efficiency) project the states \( |g\rangle, |r\rangle, \) and \( \cos(\theta)|h\rangle + \sin(\theta)|v\rangle = |l\rangle + e^{2\pi i/8} |r\rangle \), where \( n = 0, \ldots, 8 \). By displacing the holograms for \( |l\rangle (|r\rangle) \) we project arbitrary linear combinations of \( |g\rangle \) and \( |l\rangle (|r\rangle) \).
[31] Displaced plane-wave holograms allow a small leakage of unwanted states into the fiber \([18]\). This potentially explains the smaller-than-predicted Bell parameter for the nonmaximally entangled spatial-mode states \([e.g., S_{\text{exp}} = 2.28(1) \] versus the prediction \( S_{\text{pred}} = 2.35)\).
[35] Data in Fig. 2 (Fig. 3) were collected for 40 s (20 s) per projection with ~600 (~100) detected pairs/s.