LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

FACULTY OF PHYSICS

COMPUTATIONAL & PLASMA PHYSICS

Strong-field QED PIC simulation

Constantin Klier[1,2], Hartmut Ruhl[1]

[1] Fakultät für Physik, LMU München, Theresienstraße 37A

[2] IMPRS APS, Max-Planck Institut für Quantenoptik, Garching

Motivation

At laser intensities beyond $10^{25}\,\frac{\mathrm{W}}{\mathrm{cm}^2}$ QED processes will become essential to understand laser propagation and laser plasma interaction. We extend a PIC code to study this question.

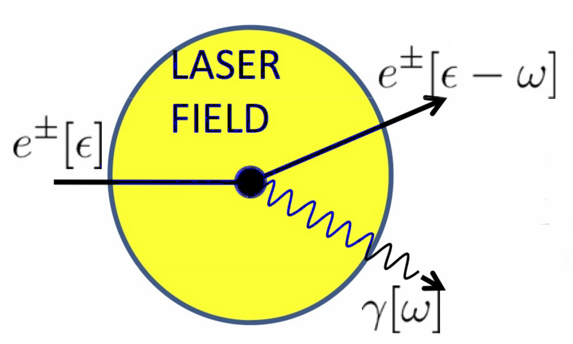
Induced processes

Strong Field processes depend on the field scalars $\mathcal{S} = (\mathbf{E}^2 - \mathbf{B}^2)/E_S^2$, $\mathcal{P} = \mathbf{E}\cdot\mathbf{B}/E_S^2$ and on the dynamical quanum parameters of the seed particle, an electron or positron with velocity $\mathbf{v}$, or a γ-photon with momentum k^μ

$$\chi = \frac{e\hbar}{m^3}\sqrt{-(F_{\mu\nu}p^\nu)^2} \quad \text{and} \quad \kappa = \frac{e\hbar}{m^3}\sqrt{-(F_{\mu\nu}k^\nu)^2} \tag{1}$$

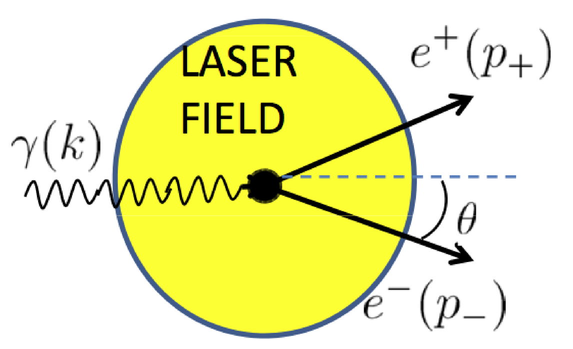
Probability rates of induced QED processes in a general external field can be expressed as $W(\chi,\mathcal{S},\mathcal{P})$. For $\chi \gg \mathcal{S},\mathcal{P}$ and $\mathcal{S},\mathcal{P} \ll 1$ processes in arbitrary external fields approach those in a constant crossed field $\mathcal{S} = \mathcal{P} = 0$ of the same strength,

$$W(\chi,\mathcal{S},\mathcal{P}) \overset{\chi\gg\mathcal{S},\mathcal{P}}{\to} W(\chi,0,0)$$



In a constant crossed field the probability rate of an electron at energy ϵ radiation a γ-photon with energy ω is

$$\frac{\mathrm{d}W_{\mathrm{rad}}}{\mathrm{d}\omega}(\omega|\epsilon) = \frac{\alpha m^2}{\epsilon^2}\Big[\int_x^\infty \mathrm{Ai}(\xi)\mathrm{d}\xi + \big(\frac{2}{y} + \kappa\sqrt{x}\big)\mathrm{Ai}'(x)\Big]_{x=\left[\frac{\kappa}{\chi(\chi-\kappa)}\right]^{2/3}}$$
$$= 1.44\frac{\alpha m^2}{\epsilon}\chi(1-0.924\chi) + O(\chi^3)$$



and of a γ-photon with energy ω converting to a pair of electron and positron is

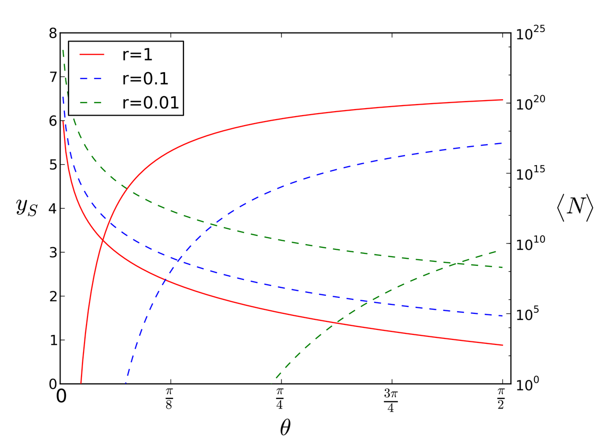
$$\frac{\mathrm{d}W_{\mathrm{pair}}}{\mathrm{d}\epsilon}(\epsilon|\omega) = \frac{\alpha m^2}{\omega^2}\Big[\int_y^\infty \mathrm{Ai}(\xi)\mathrm{d}\xi + \big(\frac{2}{y} - \kappa\sqrt{y}\big)\mathrm{Ai}'(y)\Big]_{y=\left[\frac{\kappa}{\chi(\kappa-\chi)}\right]^{2/3}}$$
$$\overset{\kappa\to 0}{\longrightarrow} 0.230\frac{\alpha m^2}{\omega}\kappa e^{-\frac{8}{3\kappa}}$$

Spontaneous pair creation

Spontaneous creation of an electron positron pair from vacuum by an electromagnetic field with invariants $\mathcal{S}$ and $\mathcal{P}$ is governed by the invariant electric field parameter $a = \sqrt{\sqrt{\mathcal{S}^2+\mathcal{P}^2} - \mathcal{S}}$ and the invariant magnetic field parameter $b = \sqrt{\sqrt{\mathcal{S}^2+\mathcal{P}^2} + \mathcal{S}}$

$$W_{\mathrm{spont}} = \frac{e^2ab}{8\pi^2}\coth\left(\frac{b}{a}\right)\exp\left[-\frac{1}{a}\right] \tag{2}$$

Particles are produced in the rest frame of the electromagnetic field, therefore can acquire a high boost factor with respect to the lab frame. This is the case if asymmetric pulses are used ([LR11], [KRLR12]). In this case $r = \frac{a_0^w\omega^w}{a_0^s\omega^s} \leq 1$.



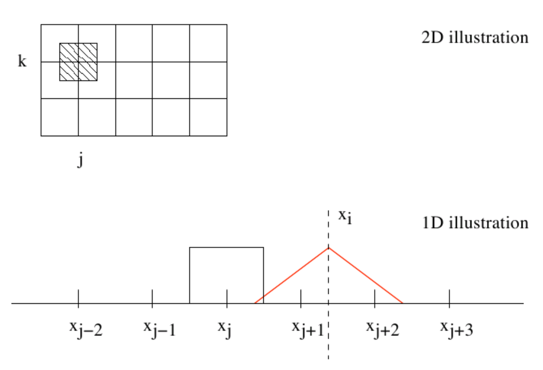
The rapidity y_S of the moving rest frame (left scale) and expected number of pairs (right scale) for $r = 1,\ 0.1,\ 0.01$ at fixed $a_0^s\omega^s/m = 1$.

PIC-Method

The kinetic approach leads to Boltzmann-type equations for charged particle and hard photon distribution functions

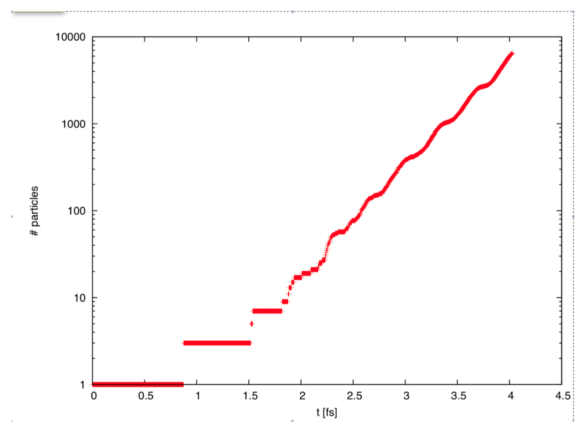
$$(\partial_t + \mathbf{v}\cdot\partial_{\mathbf{x}} + \mathbf{F}\cdot\partial_{\mathbf{p}})\, f_\pm(\mathbf{x},\mathbf{p},t) = \int \mathrm{d}^3k\, \frac{\mathrm{d}W^{\mathbf{E},\mathbf{B}}_{\mathrm{rad}}}{\mathrm{d}^3k}(\mathbf{k},\mathbf{p}+\mathbf{k}) f_\pm(\mathbf{x},\mathbf{p}+\mathbf{k},t) - f_\pm(\mathbf{x},\mathbf{p},t)\int \mathrm{d}^3k\, \frac{\mathrm{d}W^{\mathbf{E},\mathbf{B}}_{\mathrm{rad}}}{\mathrm{d}^3k}(\mathbf{k},\mathbf{p}) + \int \mathrm{d}^3k\, \frac{\mathrm{d}W^{\mathbf{E},\mathbf{B}}_{\mathrm{pair}}}{\mathrm{d}^3p}(\mathbf{k},\mathbf{p}) f_\gamma(\mathbf{x},\mathbf{k},t) \tag{3}$$

$$(\partial_t + \frac{c^2\mathbf{k}}{\omega}\frac{\partial}{\partial\mathbf{x}})\, f_\gamma(\mathbf{x},\mathbf{k},t) = \int \mathrm{d}^3p\, \frac{\mathrm{d}W^{\mathbf{E},\mathbf{B}}_{\mathrm{rad}}}{\mathrm{d}^3k}(\mathbf{k},\mathbf{p})\big[f_+(\mathbf{x},\mathbf{p},t) + f_-(\mathbf{x},\mathbf{p},t)\big] - f_\gamma(\mathbf{x},\mathbf{k},t)\int \mathrm{d}^3p\, \frac{\mathrm{d}W^{\mathbf{E},\mathbf{B}}_{\mathrm{pair}}}{\mathrm{d}^3p}(\mathbf{k},\mathbf{p}) \tag{4}$$



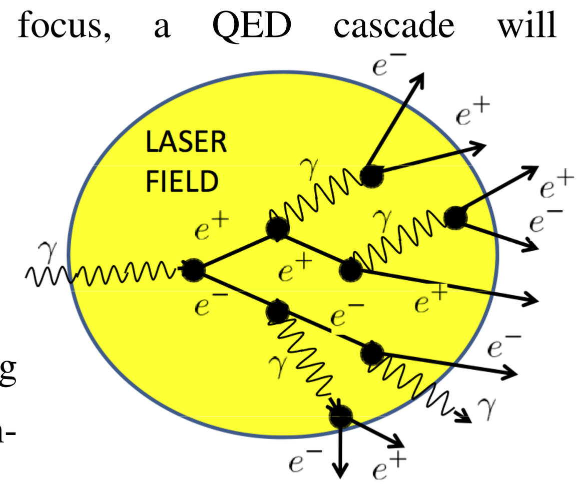
Expanding the distribution functions in quasiparticles $f_\gamma(\mathbf{x},\mathbf{k},t) = \frac{n_\gamma}{N_\gamma(0)}\sum_{i=1}^{N_\gamma(t)}\phi_\gamma(\mathbf{x}-\mathbf{x}_i^\gamma)\delta^3(\mathbf{k}-\mathbf{k}_i)$ and $f_\pm(\mathbf{x},\mathbf{p},t) = \frac{n_\pm}{N_\pm(0)}\sum_{i=1}^{N_-(t)}\phi_-(\mathbf{x}-\mathbf{x}_i^\pm)\delta^3(\mathbf{p}-\mathbf{p}_i^\pm)$ one obtains a PIC code for a plasma of electrons, positrons and photons interacting by radiadion process and pair creation.

QED cascades



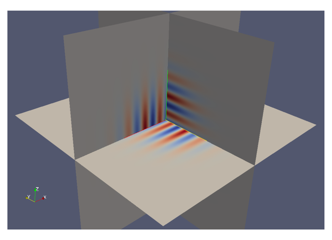
Total particle number in simulation, starting from one electron, rotating electric field at intensity $I_0 = 1.0\times10^{25}\,\frac{\mathrm{W}}{\mathrm{cm}^2}$

Starting from a single electron in the focus, a QED cascade will develop

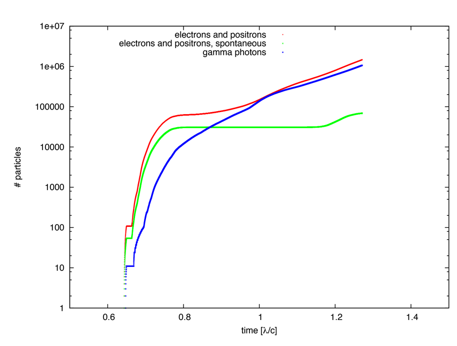


For a homogenous rotating electric field $\mathbf{E} = (E_0\cos\omega t, E_0\sin\omega t, 0)$, an analytic estimate ([FNMK10]) gives that the number of electron produced by a QED cascade scales as $N(t) = N_0 e^{\Gamma t}$ with $\Gamma \propto E^{1/4}\sqrt{\omega}$.

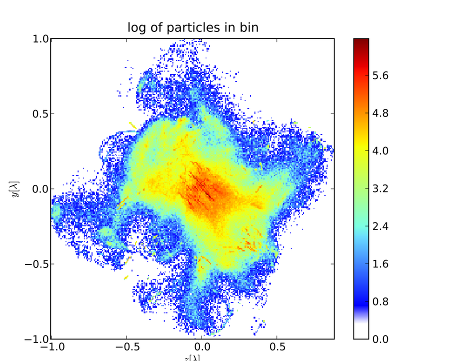
Laser collision at $I_0 = 5\times10^{26}\,\frac{\mathrm{W}}{\mathrm{cm}^2}$



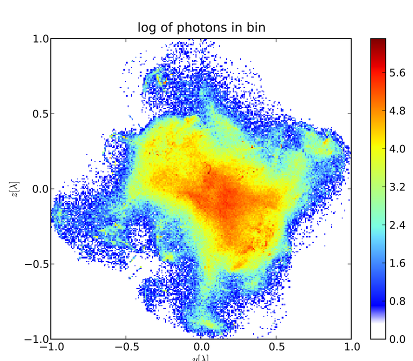
Collision of 6 pulses: along each axis, x, y, z two pulses are colliding. Setup at start of simulation 1μm from focus.



Evolution of particle and photon numbers



Density of particles, projection in $y-z$ plane



Density of photons, projection in $y-z$ plane

References

[FNMK10] A. M. Fedotov, N. B. Narozhny, G. Mourou, and G. Korn, *Limitations on the attainable intensity of high power lasers*, Phys. Rev. Lett. **105** (2010), 080402.

[KRLR12] C. Klier, H. Ruhl, L. Labun, and J. Rafelski, *Laser collisions and spectra of particles*, in preparation (2012).

[LR11] L. Labun and J. Rafelski, *Spectra of particles from laser-induced vacuum decay*, Physical Review D **84** (2011), no. 3, 033003.