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Trident pair production in a constant-crossed field

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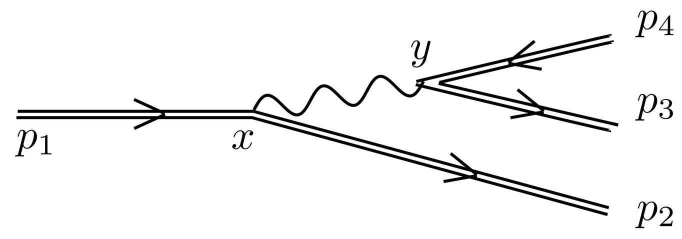
Introduction

Encouraging experimental activity such as the development of high-intensity laser facilities that can reach intensities of up to 10^{22} Wcm^{-2} [1], as well as the demonstration of massive creation of electron-positron pairs when thin metal foils are irradiated by such lasers, has led to a heightened interest in developing computer codes that can model such processes [2,3]. Although codes already exist with the lowest-order fundamental QED processes included, it is not entirely clear what influence the inclusion of higher-order processes will have on the evolution of an electron-positron plasma in a laser field. Current work is focused on one such higher-order process, trident pair creation, in which a Compton-scattered photon creates a pair. The aim of this work is to determine the difference between simply integrating the two lowest-level processes of Compton scattering and pair creation, and the full QED process. It is believed that the different spin and polarisation structure, as well as coherent and incoherent channels will increase insight into this question and perhaps lead to generalisations of even higher-order processes.

Constant-crossed field

The trident process has been most recently calculated in complicated laser backgrounds [4,5] and evaluated numerically. We calculate the trident process in a very useful and common approximation to an intense background electromagnetic field, known as the constant-crossed field limit, in which the electric and magnetic fields are taken as perpendicular and constant. In this limit, the electromagnetic relativist invariants $\mathscr{F} = \mathscr{G} = 0$ and χ is the only Lorentz-invariant scalar. Following [6], considering the probability for processes as a function of these three parameters $W(\chi, \mathscr{F}/E_{\text{cr}}^2, \mathscr{G}/E_{\text{cr}}^2)$, (where $E_{\text{cr}} = m^2/e$ with $\hbar = c = 1$ here on in), when $\mathscr{F}$ and $\mathscr{G}$ are small, through a Taylor-expansion of probabilities in these parameters, one can see that the case of a constant-crossed field $W(\chi, 0, 0)$ is a valid approximation to probability in an arbitrary constant field when $\mathscr{F}/E_{\text{cr}}^2, \mathscr{G}/E_{\text{cr}}^2 \ll 1, \chi$. Our calculations are a more complete derivation of the approximation calculated in [7].

Calculational method



We calculate the trident process above, (where solid double-lines indicate dressed fermions and wavy lines photons), with the following integral:

$$S_{fi} = e^2 \int d^4x\, d^4y\, \overline{\psi}_2(x)\gamma^\mu\psi_1(x)G_{\mu\nu}(x-y)\overline{\psi}_3(y)\gamma^\nu\psi_4^+(y) - (p_2 \leftrightarrow p_3) = \bar{S}_{fi} - \overset{\times}{S}_{fi} \tag{1}$$

where $\bar{S}_{fi}$ and $\overset{\times}{S}_{fi}$ define "direct" and "exchange" terms and the fermion wavefunctions ψ are those given by the Volkov solution in a constant-crossed field:

$$\psi_r(p) = \left[1 + \frac{e\not{\varkappa}\not{A}}{2\varkappa p}\right]\frac{u_r(p)}{\sqrt{2p^0}}\mathrm{e}^{iS(p)}; \quad \psi_r^+(p) = \left[1 - \frac{e\not{\varkappa}\not{A}}{2\varkappa p}\right]\frac{v_r(p)}{\sqrt{2p^0}}\mathrm{e}^{iS(-p)}; \quad S = -px - \int_0^\varphi d\phi \left(\frac{e(pA)}{\varkappa p} - \frac{e^2A^2}{2(\varkappa p)}\right),$$

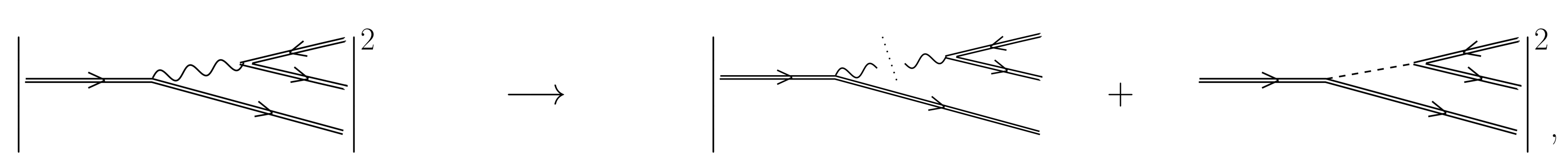
$\varkappa$ is the external field wave-vector and A its vector potential, and $G^{\mu\nu}(x-y)$ is the undressed photon propagator. Constant-crossed field wavefunctions can be Fourier-transformed into Airy functions. Applying this to Eq. (1), we acquire:

$$\bar{S}_{fi} = \frac{(2\pi e)^2}{2\varkappa\delta p}\int dr\, ds\, \delta^{(4)}(\Delta p - (r/2\varkappa\delta p + s)\varkappa)\, \Gamma^\mu(r)\, \frac{1}{r - r_* + i\varepsilon}\, \Delta_\mu(s), \tag{2}$$

where r and s are the new Fourier variables (equivalent to photon number when a plane-wave background is used instead), $r_* = (\delta p)^2/2\varkappa\delta p$, $\delta p = p_2 - p_1$ and $\Gamma^\mu(r)$, $\Delta_\mu(s)$ are functions of spinors and Airy functions. Now *Plemelj's theorem* allows one to expand the propagator in terms of on- and off- shell (or "two-step" and "one-step") parts:

$$\int dr\, \frac{f(r)}{r - r_* + i\varepsilon} = f(r_*) + \text{P. V.} \int dr\, \frac{f(r)}{r - r_*}, \tag{3}$$

where P. V. stands for "principal value". We can show this schematically as:



where the dotted line indicates the photon is on-shell and the dashed line off-shell. As a result, we can write the total rate R in the form:

$$R = \frac{\alpha^2}{p_1^0(2\pi)^3L_\phi}\prod_{j=2,3}\int \frac{d^2p_j^\perp}{(2\pi)^3}\frac{dp_j^-}{2p_j^-}\frac{\prod_{i=2}^4 2p_i^0}{2p_4^-}\mathrm{Tr}\left|\bar{S}^{(1)} - \overset{\times}{S}^{(1)} + \bar{S}^{(2)} - \overset{\times}{S}^{(2)}\right|^2,$$

$$\boxed{R = R^{(11)} + R^{(22)} + R^{(12)} + R^{(21)}} \qquad \boxed{R^{(jj)} = R^{(jj)}(-,-) + R^{(jj)}(\times,\times) + R^{(jj)}(-,\times) + R^{(jj)}(\times,-)},$$

where $R^{(11)}$ represents purely one-step terms, $R^{(22)}$ purely two-step terms, and we have exemplified the various possibilities for cross-terms due to exchange symmetry in these parts.

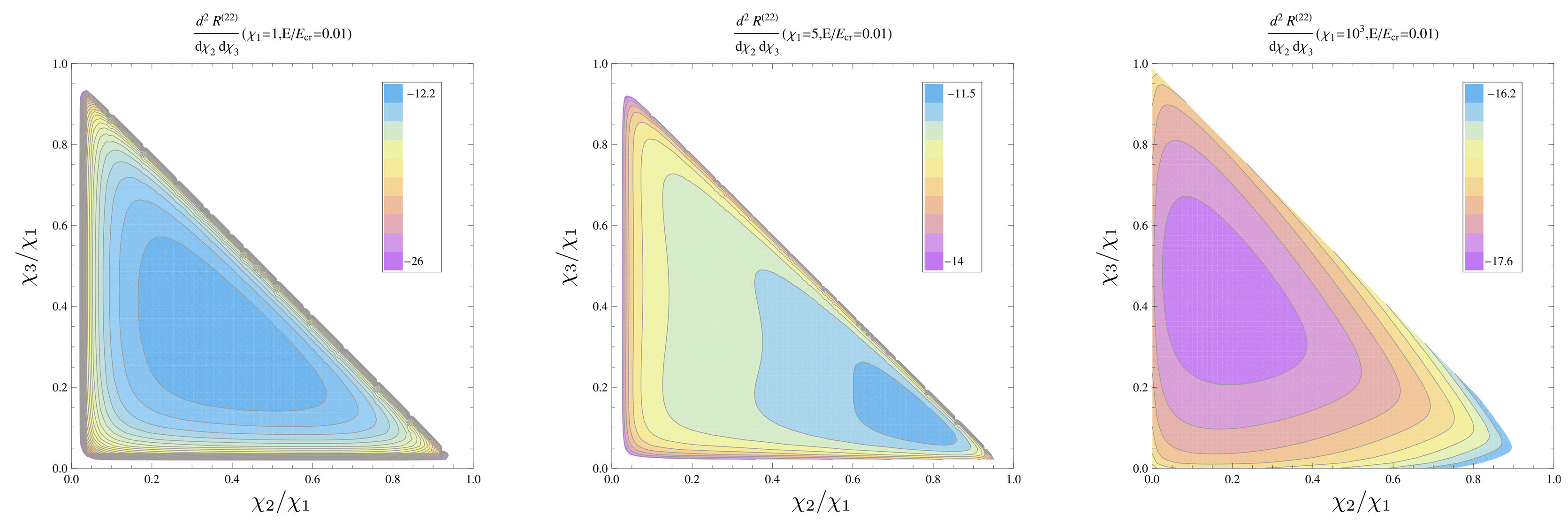
Comparison to lowest-order rates

To compare the two-step contribution to the typical approximation of integrating over lowest-order Compton scattering and pair creation rates we can define such a rate $R_{\gamma e}$:

$$R_{\gamma e} = T\int \frac{d^2k^\perp dk^-}{(2\pi)^3}\frac{(k^0)^2}{(k^-)^2}\frac{d^3R_\gamma(k)}{dk^3}R_e(k)., \tag{4}$$

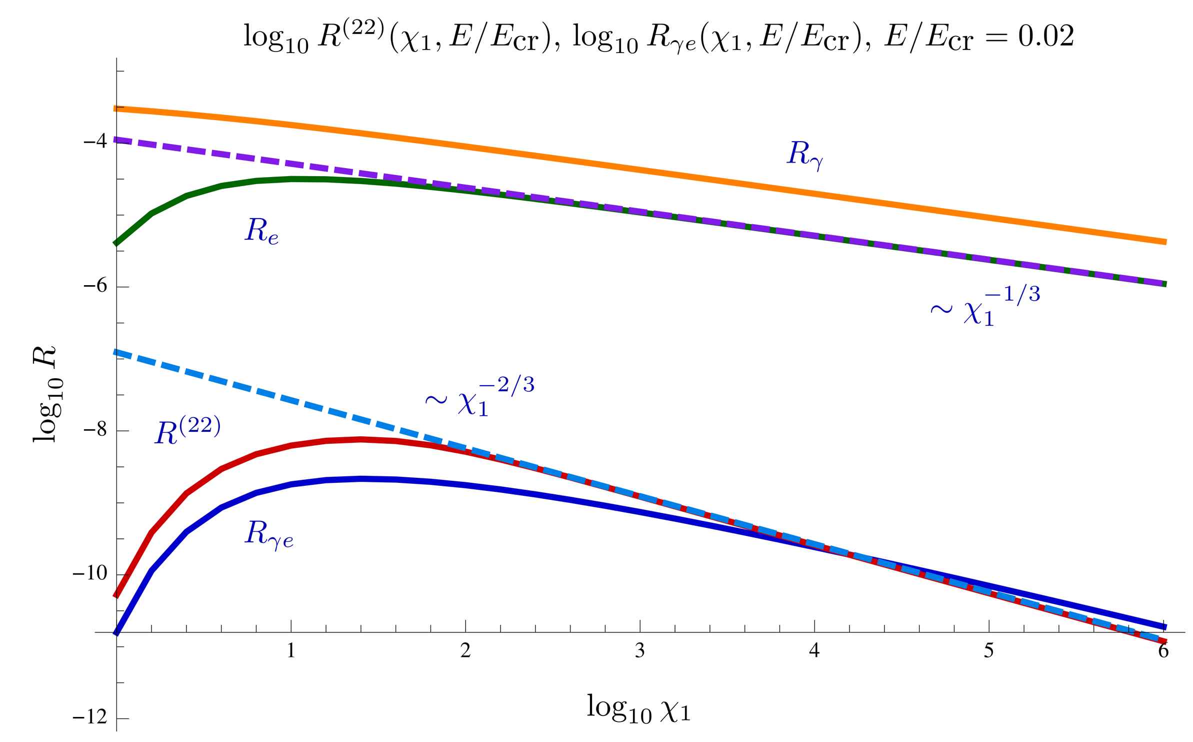
where R_γ and R_e are the rates due to Compton scattering and pair creation in a constant-crossed field respectively and T is the time in the external field. The factor of T is characteristic of a two-step process. One can show $R^{(22)} \sim T$ and $R^{(11)} \sim 1$.

Differential rate (two-step)



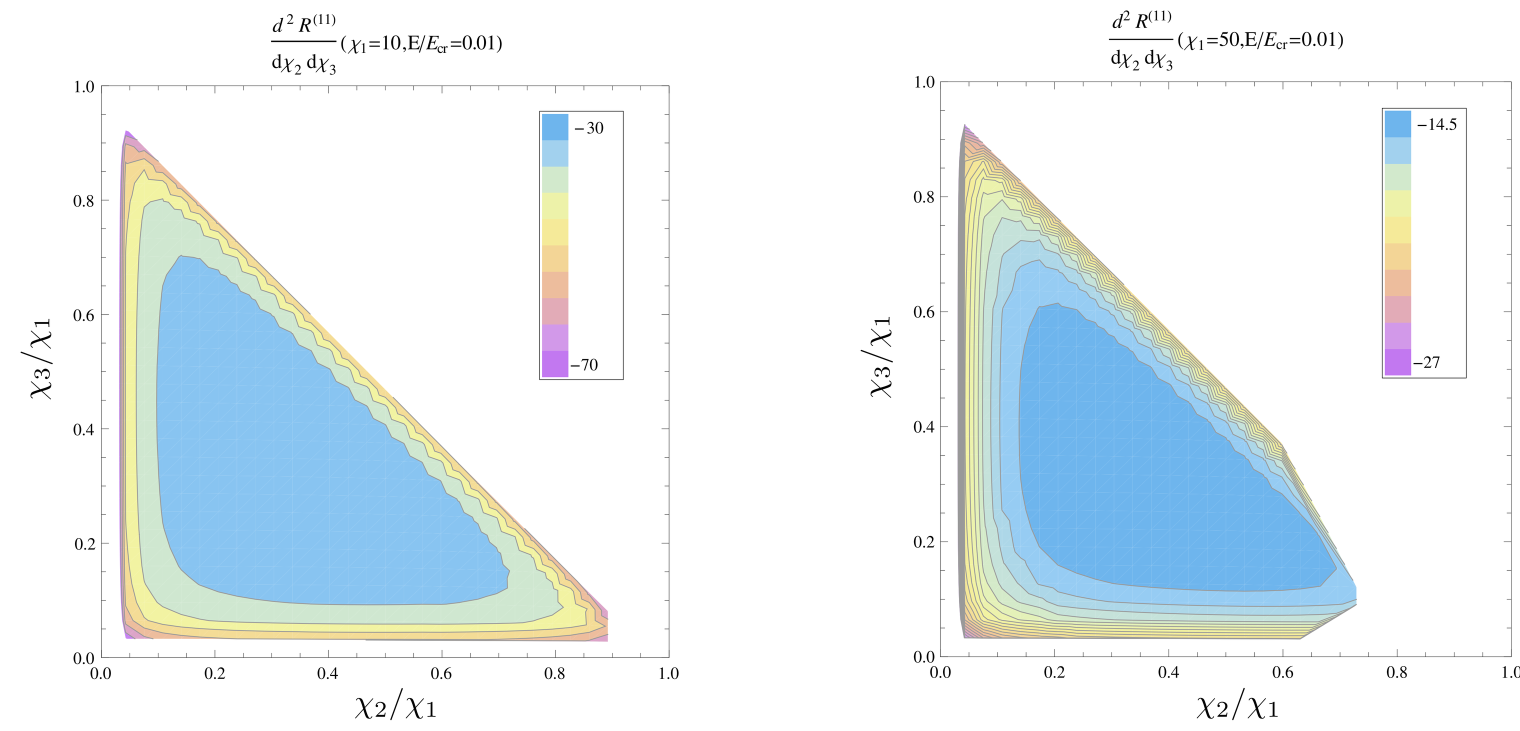
The differential rates are plotted as a function of the so-called "quantum non-linearity" parameter $\chi_i = e\hbar\sqrt{|F_{\mu\nu}p_i^\nu|^2}/m^3c^4$, of the two outgoing electrons (T has been set to the Compton-time m). At larger χ_1, it seems a larger fraction of the initial energy stays with the initial electron.

Total rate (two-step)



It can be seen that the rate for the two-step process $R^{(22)}$, is potentially higher than an integration of the lower-level processes. It also seems that the rate is suppressed more at higher χ_1. Encouraging is the asymptotic behaviour being the square of the asymptotes of the sub-processes $\chi_1^{-2/3} = (\chi_1^{-1/3})^2$.

Differential rate (one-step)



These are preliminary results. Unlike for the two-step process, it appears that it is more likely that the initial electron give more than half its energy to the pair created via a virtual photon.

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