Strong-Field QED Processes in Short Laser Pulses

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and Burkhard Kämpfer

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Introduction

- Strong Field QED: Modification of the vacuum structure and scattering amplitudes by strong electromagnetic fields which are provided by a high-intensity laser

$$I_{\mathrm{L}} > 10^{18}\ \mathrm{W/cm^2} \longrightarrow \text{relativistic optics}$$

- Most of the high-intensity optical lasers ($> 100\ \mathrm{TW}$) are short pulse lasers e.g. DRACO (20 fs), ELI (15 fs), PFS (5 fs), PEnELOPE (30 fs), …..
- only a few optical cycles

- A proper description of the finite pulse length is necessary
- Field inhomogenieties include:
  - Finite temporal pulse duration
  - Finite frequency bandwidth
  - Temporal variation of the intensity
  - Light-front structure (the laser phase) $\phi \propto x^+ = t + z$

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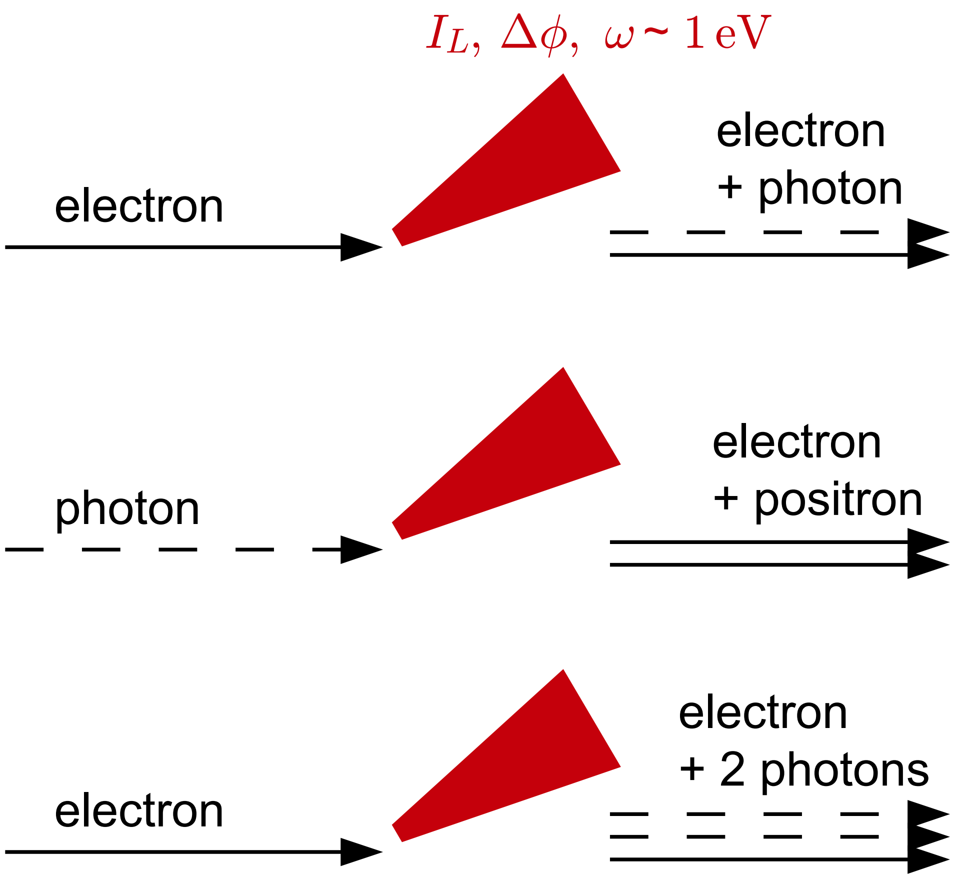
Introduction: Processes



Non-linear one-photon Compton

Narozhny and Fofanov, JETP 83, 14 (1996)
Boca and Florescu, PRA 80, 053403 (2009)
Seipt and Kämpfer, PRA 83, 022101 (2011)

Non-linear Breit-Wheeler

Narozhny and Fofanov, LasPhys 7, 141 (1996)
Heinzl, Ilderton, Marklund, PLB 692, 250 (2010)
Nousch, Seipt, et al, PLB 715, 246 (2012)

Non-linear two-photon Compton

Seipt and Kämpfer, PRD 85, 101701 (2012)
Mackenroth and Di Piazza, arXiv:1208.3424

Further processes: One-photon Annihilation, Trident Pair Production, …

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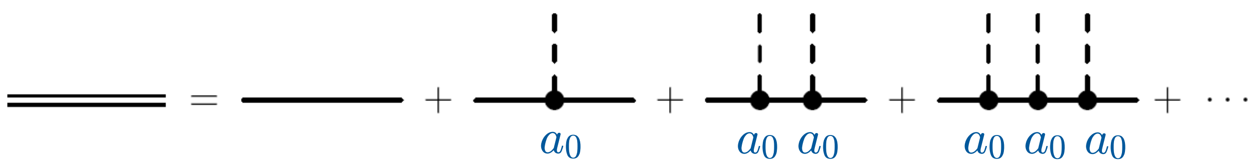
Strong-Field QED: Reiss-Nikishov-Ritus Method

- laser field is classical background field
- description of the laser pulse $$A_\mu = A\, g(\phi)\, \mathrm{Re}\left\{\epsilon_\mu(\xi) e^{i(\phi+\phi_{CE})}\right\}$$
- dimensionless laser amplitude $$a_0 = \frac{eA}{m}$$
- Furry picture: non-perturbatively laser dressed electron states

$$(i\partial\!\!\!/ - eA\!\!\!/ - m)\Psi(x) = 0$$

Volkov states: $\Psi_p(x) = E_p(x) u_p$



Laser pulse is taken into account exactly on the tree level
Details on the laser pulse are encoded in $E_p(x)$

Dirac-Volkov propagator: $$\mathcal{G}(y,x) = \int \frac{d^4p}{(2\pi)^4} E_p(y) G_0(p) \bar{E}_p(x)$$

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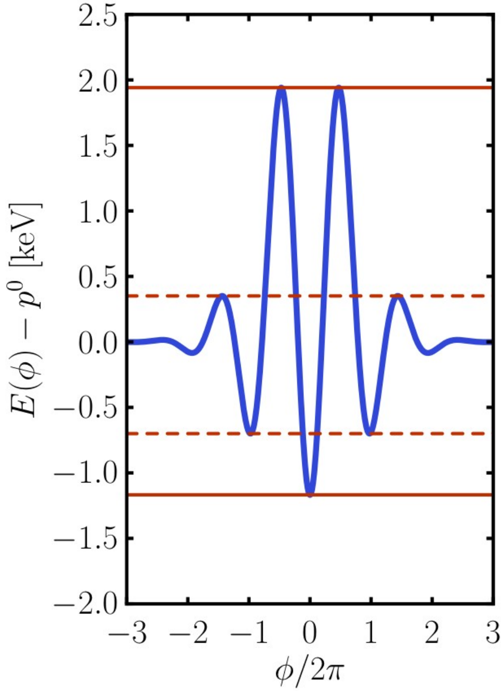
Properties of the Volkov Matrix Functions

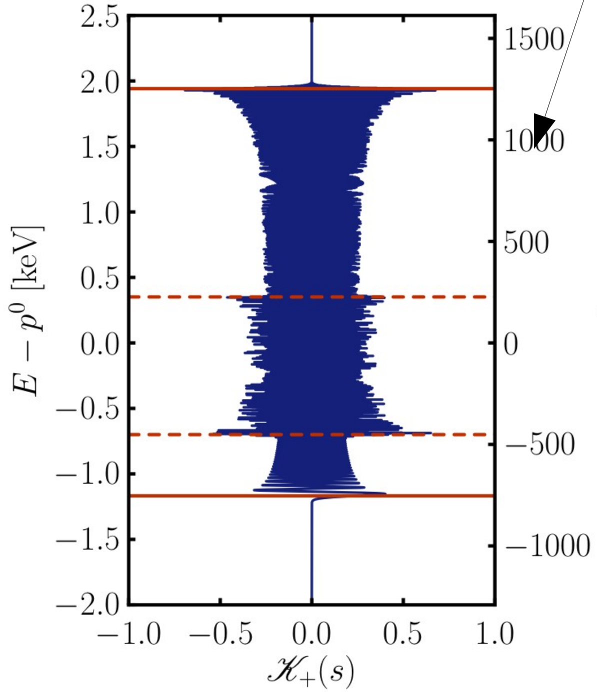
spectral representation $E_p(x) = \int \frac{ds}{2\pi} e^{-i(p+sk)\cdot x} K(s)$

laser pulse properties: $a_0,\ g,\ \Delta\phi,\ \phi_{\mathrm{CE}},\ \xi$

class. trajectory

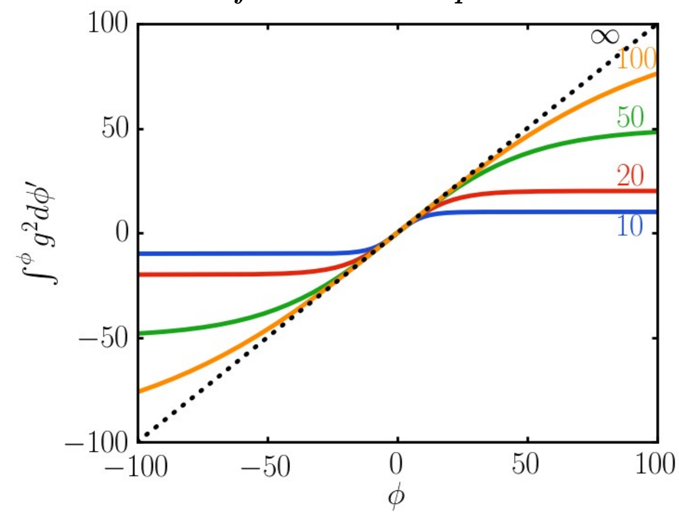


Volkov state occupation



different asymptotics for IPW and PPW

$$\Psi_{future} \sim \Psi_{past}\, e^{-i\Delta f}$$



$$\Delta f \sim a_0^2 \Delta\phi$$

IPW: phase shift diverges → quasi-momentum

$$q = p + \frac{m^2 a_0^2}{4k\cdot p} k$$

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One-photon Compton Scattering in an Infinite Plane Wave

Collision of a weak IPW laser field with an electron:
Inverse Compton/Thomson scattering → X-ray production

The emitted frequency is the Doppler up-shifted laser frequency (low energy):

$$\omega' = \chi(\theta)\,\omega \quad \xrightarrow{\theta=0} \quad \chi = 4\gamma^2$$

High-energy electrons: recoil

$$\omega' = \chi(\theta, \tfrac{2\gamma\omega}{m})\,\omega \quad \xrightarrow{\theta=0} \quad \chi < 4\gamma^2$$

In a strong laser field: interactions with many photons

$$\omega'_n = n\,\chi(\theta, n\tfrac{2\gamma\omega}{m}, a_0)\,\omega \quad \xrightarrow{\text{low energy, } \theta=0} \quad \omega'_n = \frac{n}{1+a_0^2/2} 4\gamma^2\omega$$

interaction with n photons → harmonics
modified boundary conditions: quasi-momentum → intensity dependent red-shift

line spectrum in an IPW

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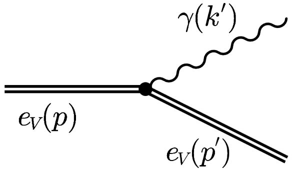
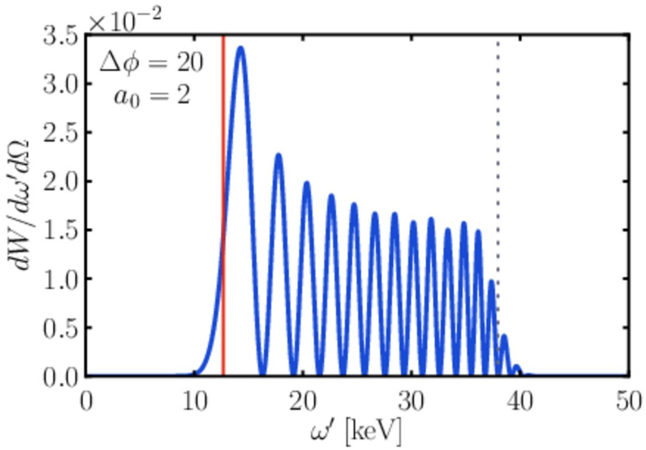
Non-linear one-photon Compton Scattering

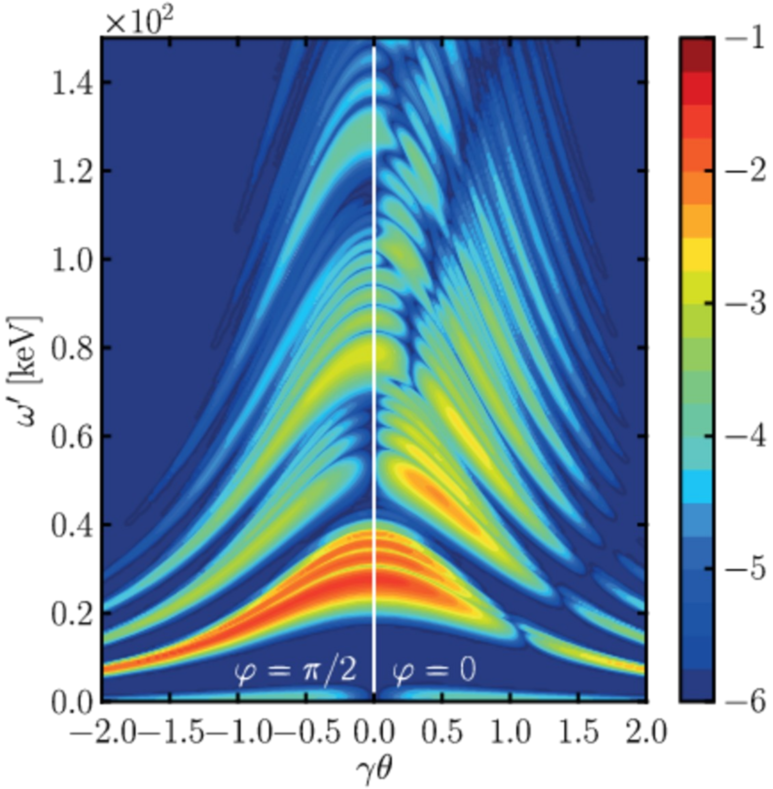
$$S = \langle \boldsymbol{p}' r'; \boldsymbol{k}' \lambda' | \hat{S}[A] | \boldsymbol{p} r \rangle$$



$$S = (2\pi)^4 \int \frac{d\ell}{2\pi} \delta^4(p + \ell k - p' - k') \mathrm{M}(\ell)$$

$$dW = \frac{1}{2p^+} |S|^2 d\Pi$$





SPA: $N = a_0^2 \Delta\phi \dfrac{\nu[g]}{4\pi}$

Narrow peaks of IPW turn into broad resonances with substructure
The number N of sub-peaks is related to the asymptotic behaviour of $E_p(x)$!

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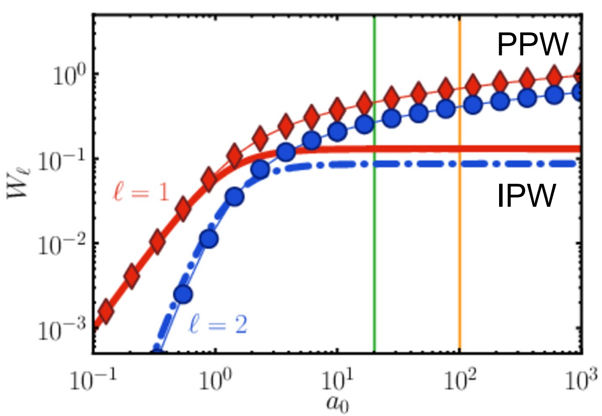
Non-linear one-photon Compton scattering: IPW vs. PPW

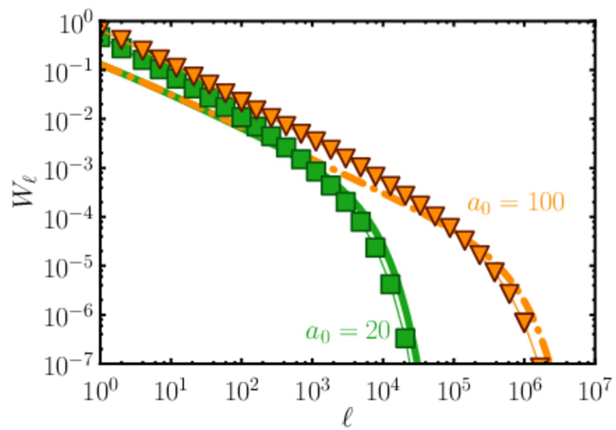
- Compare total photon yield for infinite plane wave with pulsed plane wave
- Large laser intensity $a_0 \gg 1$
- PPW: SPA: Number of subpeaks $N \gg 1 \rightarrow$ average in phase space

$$W = \sum_\ell W_\ell \qquad W^{\mathrm{IPW}}_{(\ell)} = \dot{W}^{\mathrm{IPW}}_{(\ell)} \, T_{\mathrm{eff}} \qquad T_{\mathrm{eff}} = \frac{2e^2}{m^2 a_0^2 \omega^3} \int_{-\infty}^{\infty} d\phi \, \mathcal{T}^{00}(\phi)$$

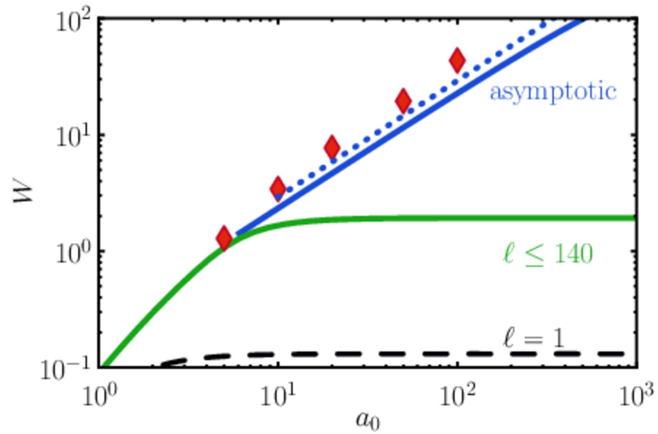
1st and 2nd harmonic



harmonics IPW vs. PPW



total photon yield



- Low harmonics: Enhancement by an order of magnitude
- High harmonics: Reduction in a PPW
- Total photon yield: Enhancement of factor 2 as compared to const. crossed field

DS and B. Kämpfer, arXiv:1111.0188

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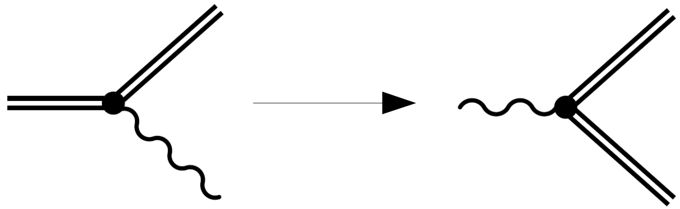
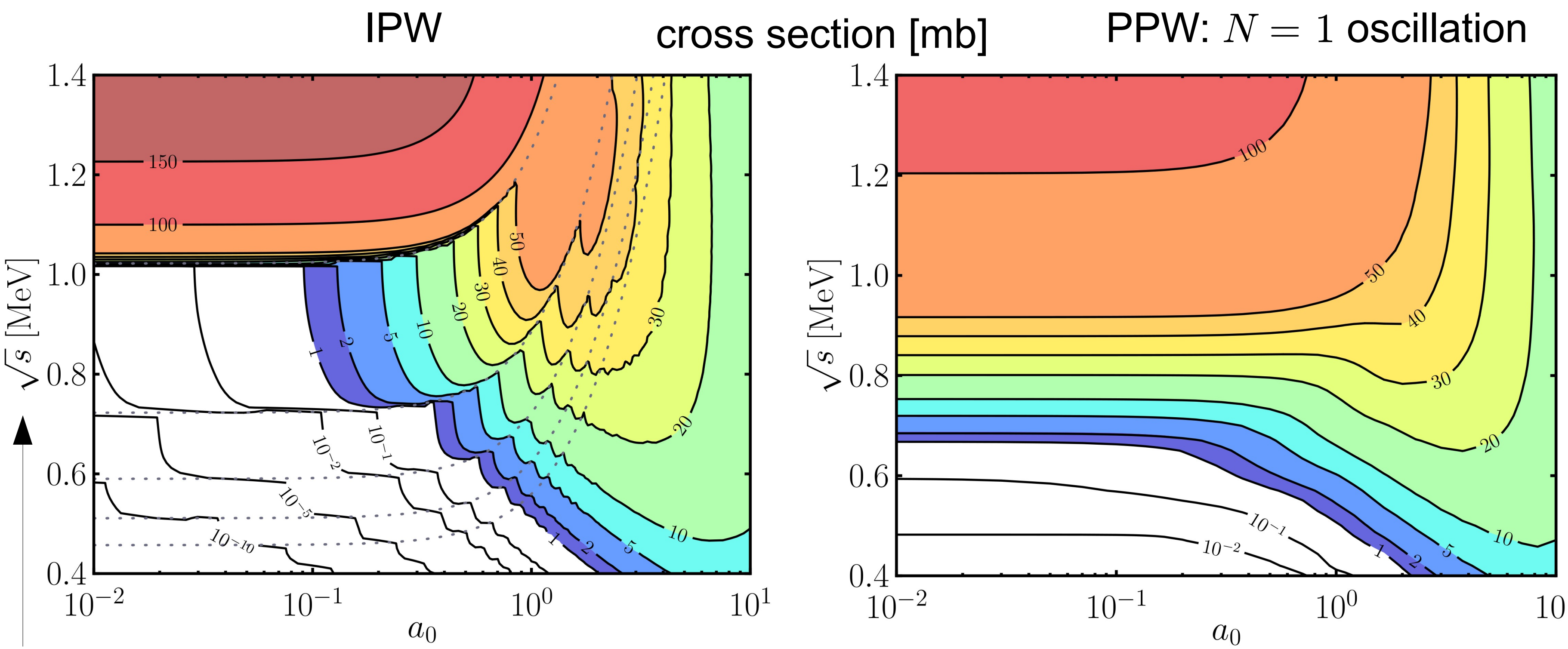
Strong-Field Pair Production

$$S = \langle \boldsymbol{p} r'; \boldsymbol{p}' r' | \hat{S}[A] | \boldsymbol{k}' \lambda' \rangle$$



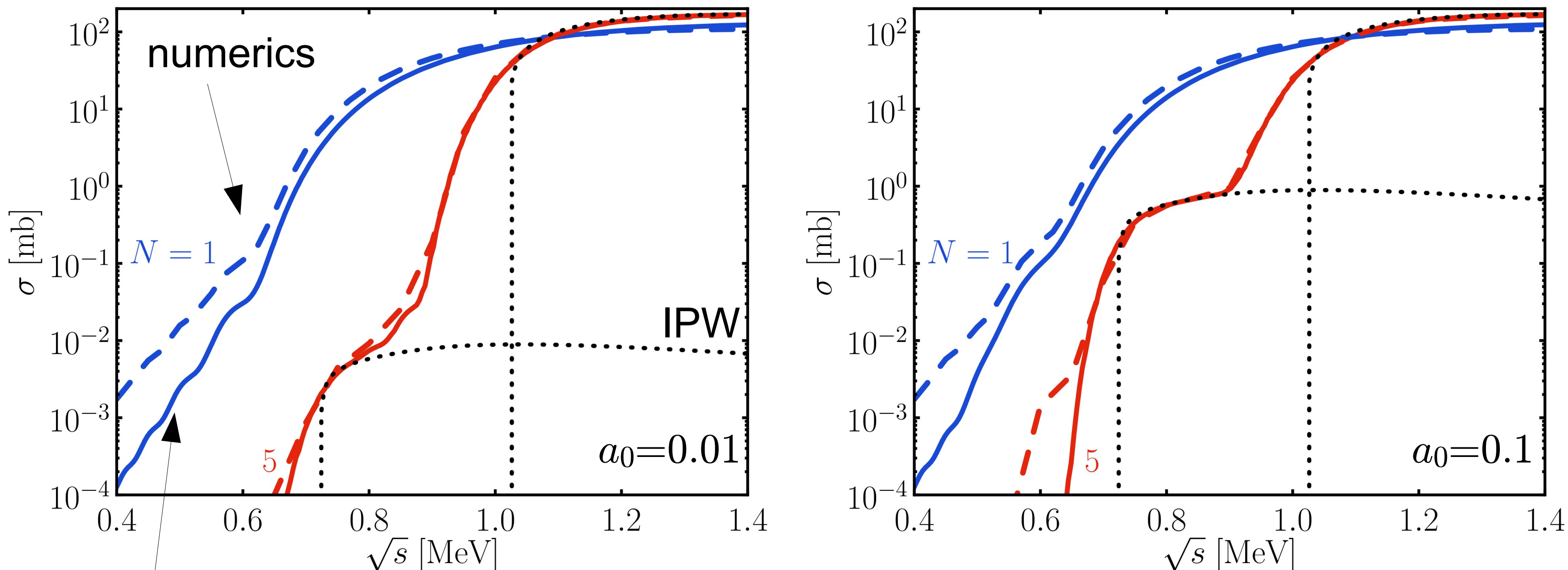


$$s = (k' + k)^2$$

Near threshold behaviour is drastically modified by short pulses

T. Nousch, DS, B. Kämpfer and A. I. Titov: PLB 715, 246 (2012)

Strong Enhancement of Pair Production Near Threshold



folding: $$\langle\sigma_n\rangle(s) = R_n \frac{\int d\ell G(\ell-1)^{2n}\sigma_n^{(0)}(\ell s)}{\int d\ell G(\ell-1)^{2n}}$$ $$R_n = \frac{\int d\phi g^{2n}(\phi)}{\int d\phi g^2(\phi)}$$

bandwidth

intensity

T. Nousch, DS, B. Kämpfer and A. I. Titov: PLB 715, 246 (2012)

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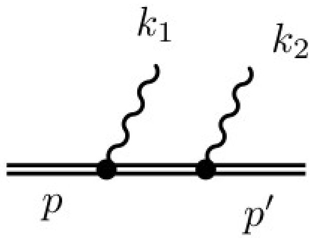
Two-Photon Compton Scattering

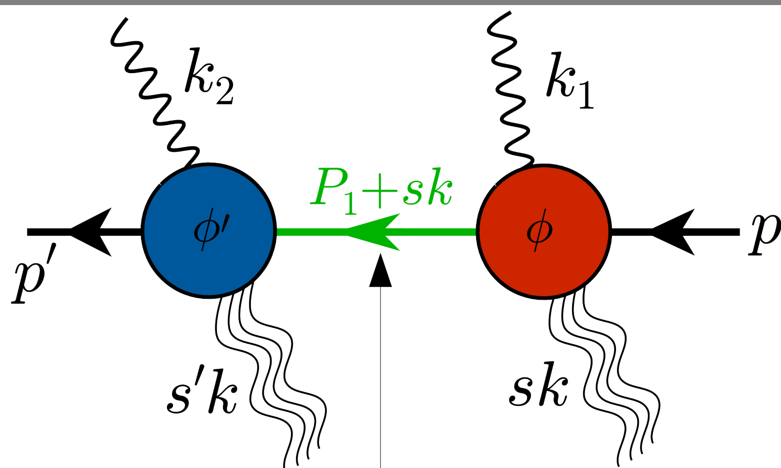
$$S = \langle \boldsymbol{p}' r'; \boldsymbol{k}_1 \lambda_1; \boldsymbol{k}_2 \lambda_2 | \hat{S}[A] | \boldsymbol{p} r \rangle$$



$+ \quad (1 \leftrightarrow 2)$

$$S = \int d^2 s d^2 \phi \, \delta(p + [s + s']k - k_1 - k_2 - p') \, \bar{u}_{p'} \, V^2_{p' P_1}(\phi', s') \, G_0(P_1 + sk) \, V^1_{P_1 p}(\phi, s) \, u_p + (1 \longleftrightarrow 2)$$

Integration over propagator pole



$P_1 = p - k_1$

real	and	virtual	intermediate electrons
$(P_1 + sk)^2 = m^2$		$(P_1 + sk)^2 \neq m^2$	

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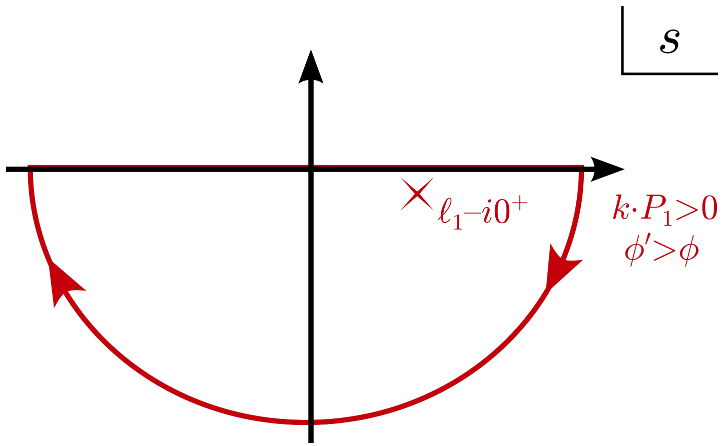
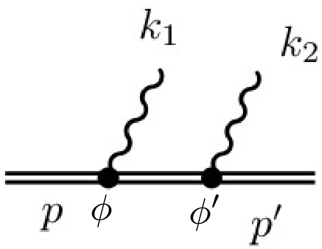
Propagator Pole Integration

$$\Delta = \int_{-\infty}^{\infty} ds\, e^{-is(\phi'-\phi)} \frac{\not{P}_1 + s\not{k} + m}{s - \ell_1 + i\,\mathrm{sign}\,(P_1^+)0^+} \qquad \ell_1 = \frac{m^2 - P_1^2}{2k\cdot P_1}$$

single pole of the propagator due to the light-front structure





„time ordering“
with respect to laser phase
or light-front time

$$\Delta = 2\pi\delta(\phi'-\phi)\not{k} - 2\pi i\Theta(\phi'-\phi)e^{-i\ell_1(\phi'-\phi)}(\not{P}_1 + \ell_1\not{k} + m)$$

instantaneous term
= light-front zero-mode
propagator

retarded term in light-front time,
second vertex at later „time“

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Numerical Results: Differential Spectra

$$dW = \frac{1}{2p^+} |S|^2 d\Pi \longrightarrow \frac{d^6 W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2}$$

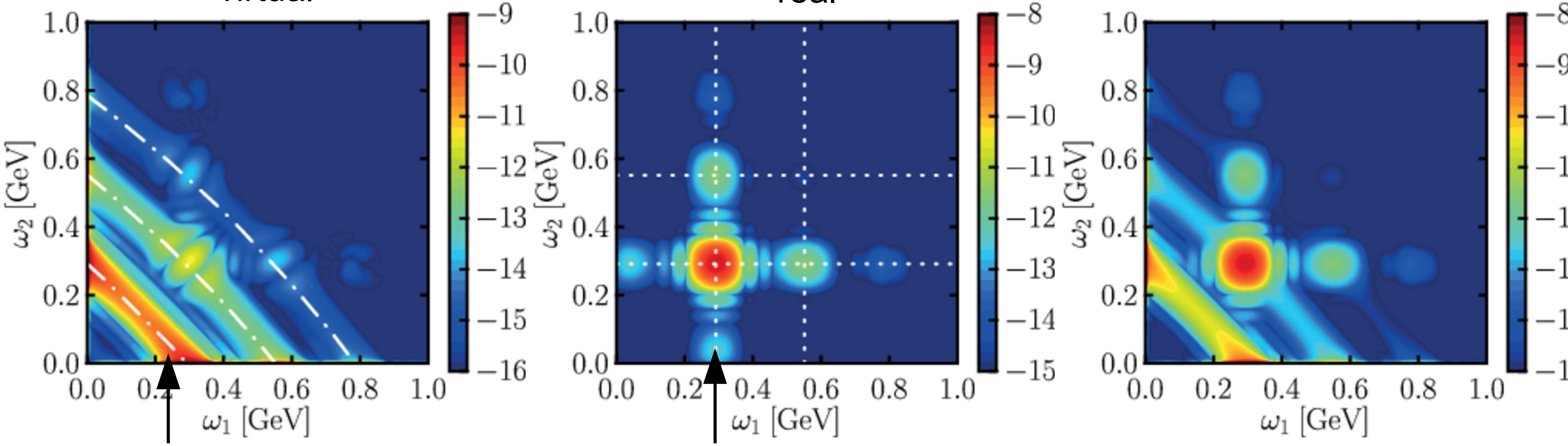
$a_0 = 0.1$

9 fs FWHM

off-shell virtual

on-shell real

total



IPW harmonics

Oleinik resonances

finite on-shell part

real on-shell part diverges in IPW, regularized by finite pulse length

$\gamma = 10^4$

$\theta_{1,2} = 0.5/\gamma$

$\varphi_1 = 0.5\pi$

$\varphi_2 = 1.5\pi$

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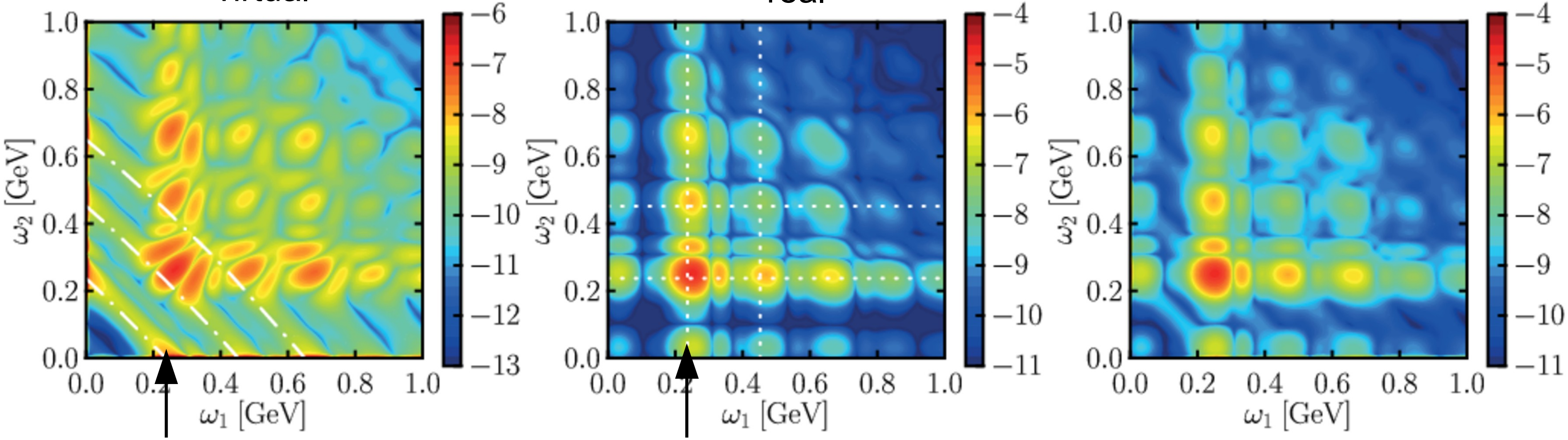
Numerical Results: Differential Spectra

$$dW = \frac{1}{2p^+}|S|^2 d\Pi \longrightarrow \frac{d^6W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2}$$

$a_0 = 1.0$

9 fs FWHM

off-shell virtual | on-shell real | total



IPW harmonics

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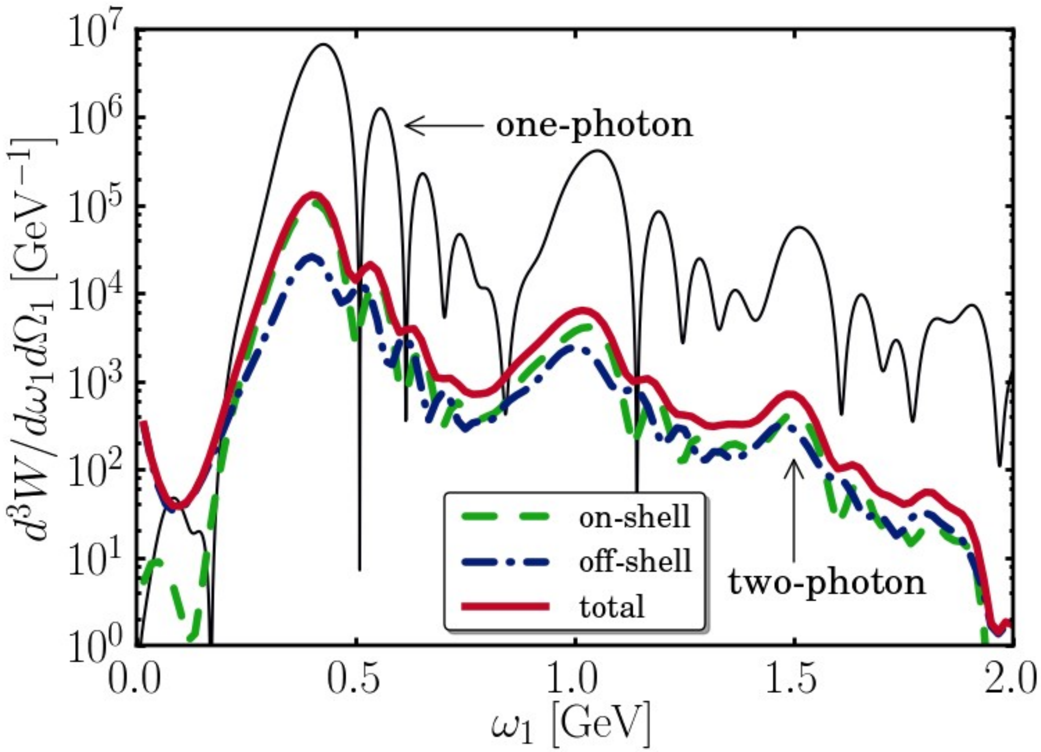
Inclusive two-photon spectra: Comparison with one-photon Compton



- integrated over second photon phase space $d\omega_2\, d\Omega_2$
- exceeds one-photon spectrum at small photon energies ω_1

$$\frac{d^3W}{d\omega_1 d\Omega_1} = \int_{\omega_{\min}} d\omega_2 \int d\Omega_2 \frac{d^6W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2}$$

- result is insensitive to value of cutoff $\omega_{\min} = 1 \dots 1000\,\mathrm{keV}$
- infrared divergent parts are the same as in perturbative QED
- cancellation of infrared divergence ensured by Bloch-Nordsieck theorem

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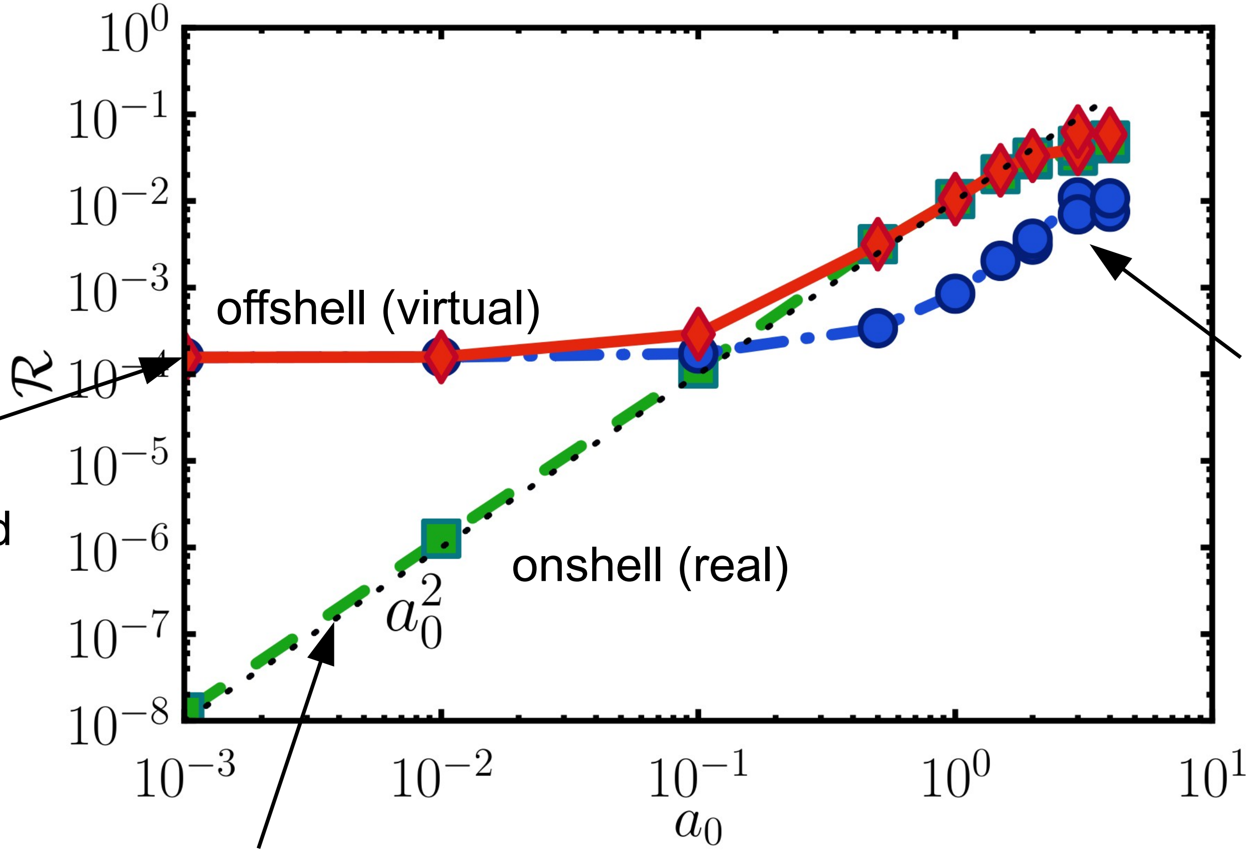
Two-Photon to One-Photon Ratio

2-photon/1-photon ratio $\mathcal{R} = \frac{dW_{(2)}}{d\Omega} \Big/ \frac{dW_{(1)}}{d\Omega} \Big|_{\theta_1=\varphi_1=0}$



correct weak-field limit

enhanced two-photon probability for strong fields

suppression of the cascade:
at least two photons are necessary

Summary

- Effect of finite laser pulse duration on strong-field QED processes: one- and two-photon Compton and Breit-Wheeler
- Emergence of sub-peaks due to ponderomotive broadening and interference
- Substantially enhanced pair production cross section near the threshold
- Finite on-shell (real) and off-shell (virtual) contributions for two-photon Compton
- Enhanced two-photon emission in strong fields

T. Nousch, DS, B. Kämpfer and A. I. Titov, PLB 715, 246 (2012)
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BACKUP

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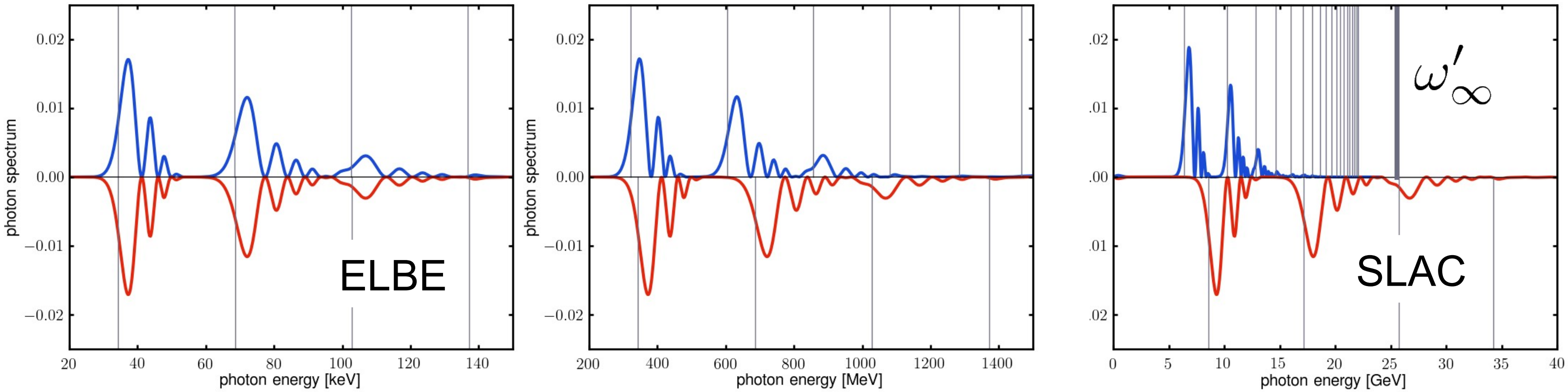
Non-Linear Compton Spectra: Quantum Effects

Quantum Calculation

- Effect of recoil
- Non-equidistant harmonics
- Maximum scattered frequency
- Spin relevant at high energy



40 MeV Increasing electron energy 25 GeV

Classical Calculation

- Self similar spectrum
- Equidistant harmonics

Both coincide at low energy

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Previous Results in Infinite Monochromatic Plane Waves

- Result of [Lötstedt/Jentschura, Phys. Rev. Lett. 103, 110404 (2009)]:

$$S = i \sum_{n=1}^{\infty} \sum_{s=-\infty}^{\infty} \frac{(2\pi)^4 e^2 m \delta^4(q_i - q_f + n\varkappa - k_b - k_c)}{2V^2 \sqrt{\omega_c \omega_b Q_i Q_f}} \times u_f^\dagger \gamma^0 \left[M_{bfc}^{n-s} \frac{\hat{f}_b + m}{p_b^2 - m_*^2} M_{ibb}^s + M_{cfb}^{n-s} \frac{\hat{f}_c + m}{p_c^2 - m_*^2} M_{icc}^s \right] u_i. \qquad (1)$$

- Propagator can go on mass shell → emission rate diverges

- Oleinik resonance infinities due to discrete level structure

$$p_{b,c}^2 = m_\star^2 \qquad p_{b,c} = q_i - k_{b,c} + sk$$

- Methods of regularization:

  - inclusion of electron decay width via imaginary mass expression
  - finite pulselength laser pulse → acts as regulartor

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Two-Photon Emission Probability

$$\frac{d^6W}{d\omega_1 d\Omega_1 d\omega_2 d\Omega_2} = \frac{\alpha^2\omega_1\omega_2}{64\pi^4(k\cdot p)(k\cdot p')}|\mathcal{M}|^2$$

differential probability

$$\mathcal{M} = \frac{1}{2k\cdot P_1}\left\{\sum_{n=0}^{2} T_n A_n(s) - i\sum_{n,l=0}^{2} U_{nl}B_{nl}(s,\ell_1)\right\} + (1\leftrightarrow 2)$$

Complete result including off-shell and on-shell contributions
Sokhotsky Weierstraß Theorem

constant Dirac currents: $T_n, U_{nl} \sim \bar{u}_{p'}\,\Gamma\,u_p$

Phase integrals over laser pulse:

$$A_n(s) = \int d\phi\, a^n(\phi)\exp\{is\phi - if(\phi)\}$$

$$a(\phi) = g(\phi)\cos(\phi)$$

$$B_{nl}(s,\ell_1) = \int d\phi\, d\phi'\, \Theta(\phi'-\phi)a^n(\phi)a^l(\phi')\exp\{i(s-\ell_1)\phi' + i\ell_1\phi - iF(\phi',\phi)\}$$

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Two-Photon Emission Probability II

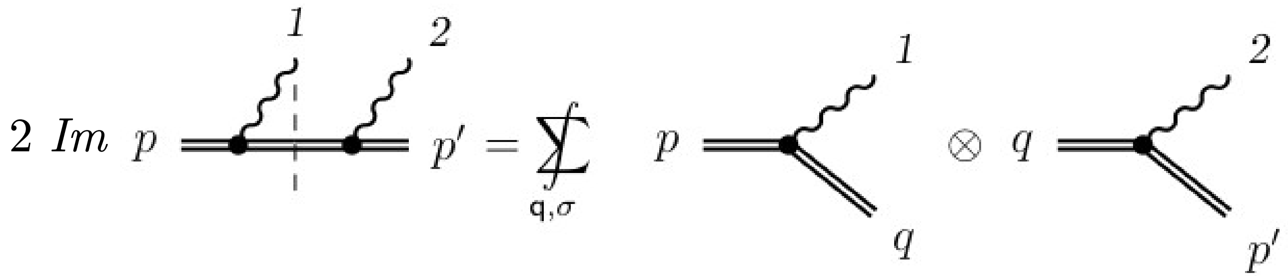
On-shell and off-shell contributions to emission probability

↓

finite resonant part ↔ Oleinik singularities

As shown by Ilderton, on-shell part is rendered finite due to causality prescription of the propagator [A. Ilderton, PRL **106**, 020404 (2010).]

Factorization of the on-shell part of the matrix element:



Realization of a Cutkosky cutting rule which is related to unitarity of the S matrix

Consequence in weak field limit:

- on-shell part scales as $a_0{}^2$, while off-shell part scales as $a_0{}^1$
- at least two photons are needed to satisfy the on-shell momentum conservation

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