Pair creation by collision of intense laser pulse with high-frequency photon beam

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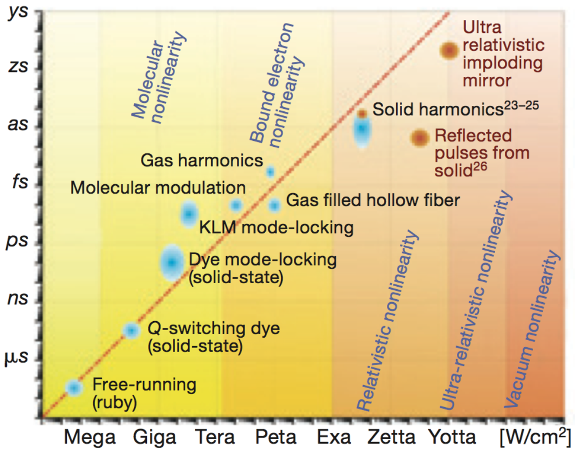
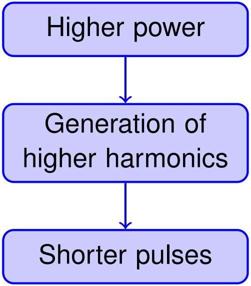
Frontiers in Intense Laser-Matter Interaction Theory
(MPQ, September 19-21, 2012)

Short pulses: perspectives

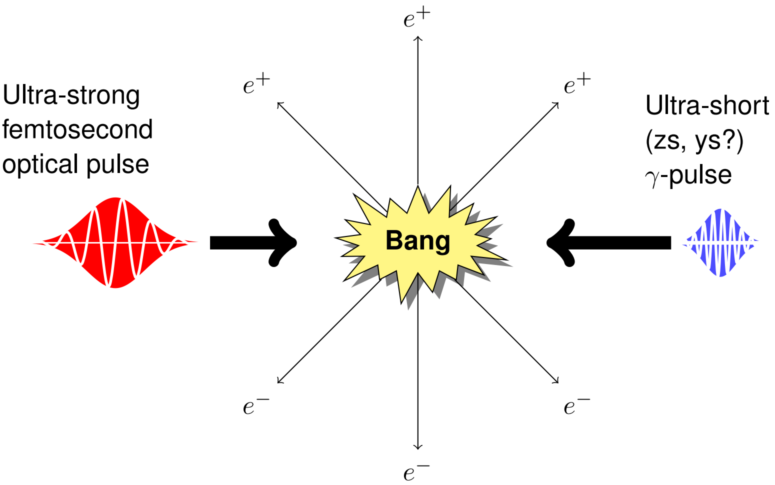


- $\text{Power} = \dfrac{\text{Energy}}{\text{Duration}}$



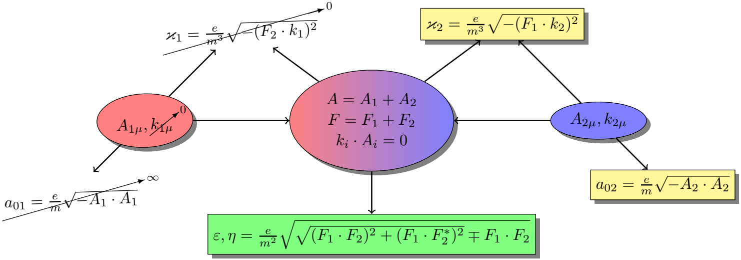
G. Mourou and T. Tajima, "More intense, shorter pulses", Sciense 331, 41 (2011).

Setup



Full problem is out of capability of existing computational methods!

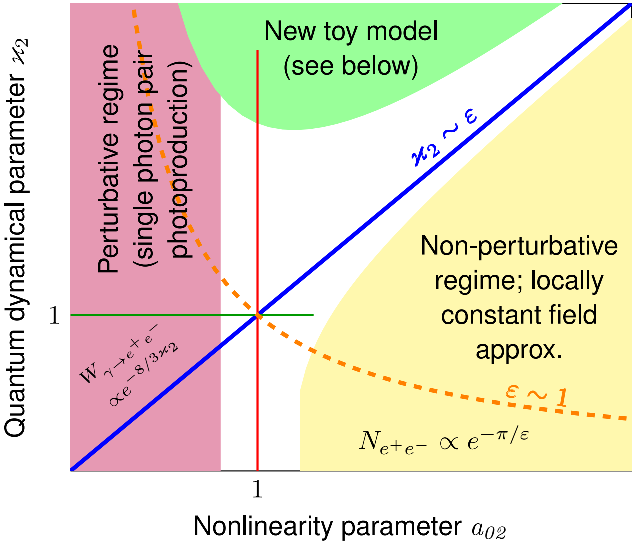
Parameters



- Explicitely, $a_{02} = eE_2/m\omega_2$ [expansion parameter for PT], $\varkappa_2 = 2eE_1\omega_2/m^3$ [dynamical quantum parameter, for subperiod pulses $\omega_2 \to \tau_2^{-1}$];
- ε and η are E and B normalized to $E_c = m^2/e$, in a RF such that $\vec{E} \perp \vec{B}$;
- Assume for simplicity that $\eta = 0$. Then

$$\varepsilon = \frac{e}{m^2}\sqrt{E^2 - B^2} = \frac{2e(E_1 E_2)^{1/2}}{m^2} = \sqrt{2a_{02}\varkappa_2}.$$

Landscape of the theory



Diagrams involved

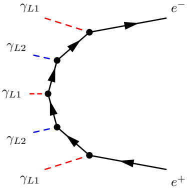


Figure: A)

Locally constant field approximation (fully non-perturbative)

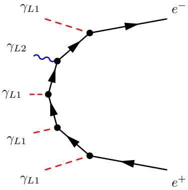


Figure: B)

Leading order of PT with respect to laser beam 2

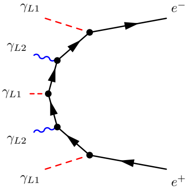
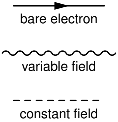


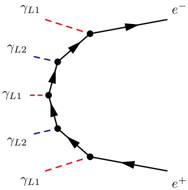
Figure: C)

Fully non-perturbative exact diagram

LEGEND:



Locally constant field approximation



$a_{02}, a_{01} \gg 1$, $\varkappa_{1,2} \ll \varepsilon, \eta$ (valid for powerful optical lasers)

Total field:

$$\vec{E} = \vec{E}_1 + \vec{E}_2, \quad \vec{B} = \vec{B}_1 + \vec{B}_2$$

Normalized invariant fields:

$$\left.\begin{matrix} \varepsilon \\ \eta \end{matrix}\right\} = \frac{e}{m^2}\sqrt{\sqrt{\left(\frac{E^2 - B^2}{2}\right)^2 + \left(\vec{E}\cdot\vec{B}\right)^2} \pm \frac{E^2 - B^2}{2}}$$

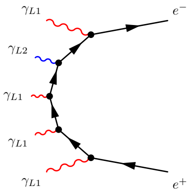
Number of created pairs:

$$N_{e^-e^+} = \int dt\, dV\, \frac{m^4 \varepsilon(\vec{r},t)\eta(\vec{r},t)}{4\pi^2} \coth\left(\frac{\pi\eta(\vec{r},t)}{\varepsilon(\vec{r},t)}\right) \exp\left(-\frac{\pi}{\varepsilon(\vec{r},t)}\right)$$

N. B. Narozhny et al, PLA 330, 1 (2004); S. S. Bulanov et al, JETP 102, 9 (2006); A. M. Fedotov, Laser Physics 19, 214 (2009).

Pair creation by photon in a plane wave

$a_{01}, \varkappa_2$ – arbitrary, $a_{02} \ll 1$ (valid for nowadays XFELs)



Auxiliary quantities:

$$s_0 = \frac{2m_*^2}{k_1 \cdot k_2} = \frac{2a_{01}\left(1 + a_{01}^2\right)}{\varkappa_2}$$

$$z = \frac{4a_{01}^2\sqrt{1+a_{01}^2}}{\varkappa_2}\sqrt{u\left(\frac{s}{s_0} - u\right)}$$

$$n_\gamma \text{ [density of } \gamma\text{'s]} \sim \frac{E_2^2}{4\pi\omega_2} = \frac{m^2\omega_2}{4\pi e^2}a_{02}^2$$

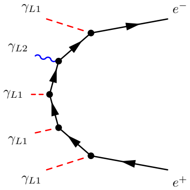
Pair creation rate (CP for definiteness):

$$W_{e^-e^+} = \frac{e^2m^2n_\gamma}{2\omega_2}\sum_{s>s_0}\int_1^{s/s_0}\frac{du}{u^{3/2}\sqrt{u-1}}\left\{J_s^2(z) + a_{01}^2\left(u - \frac{1}{2}\right)\right.$$

$$\left.\times\left[J_{s-1}^2(z) + J_{s+1}^2(z) - 2J_s^2(z)\right]\right\} \propto_{a_{01}\ll 1} a_{01}^2 \times a_{02}^2 \text{ [Breit-Wheeler]}$$

N. B. Narozhny, A. I. Nikishov, and V. I. Ritus, Sov. Phys. JETP **20**, 622 (1965).

Pair creation by photon in constant crossed wave



$a_{01} \gg 1$ (non-perturbative), $a_{02} \ll 1$ (perturbative), $\varkappa_2$ – arbitrary

Pair creation rate:

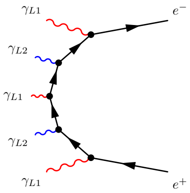
$$W_{e^-e^+} = -\frac{e^2m^2n_\gamma}{\omega_2\sqrt{\varkappa_2}}\int\limits_{(4/\varkappa_2)^{2/3}}^{\infty}\frac{d\zeta}{\zeta^{11/4}}\frac{1+2\varkappa_2\zeta^{3/2}}{\sqrt{\varkappa_2\zeta^{3/2}-4}}\,\mathrm{Ai}\,(\zeta)$$

Further limiting cases:

$$W_{e^-e^+} \sim \begin{cases} 0.23\dfrac{e^2m^2}{\omega_2}\varkappa_2 e^{-8/3\varkappa_2}, & \varkappa_2 \ll 1 \quad \text{[quasiclassics];} \\ 0.38\dfrac{e^2m^2}{\omega_2}\varkappa_2^{2/3}, & \varkappa \gg 1 \quad \text{[essentially quantum].} \end{cases}$$

A. I. Nikishov and V. I. Ritus, Sov. Phys. JETP **19***, 529 (1964); ibid* **20***, 757 (1965).*

Pair creation by variable electric field



- $E_1 = E_2 = E,\ \omega_1 = \omega_2 = \omega$ (standing wave); $\varkappa \ll \varepsilon \ll 1$
- $E(z,t) \to E(t),\ B(z,t) \to 0$ (ignoring inhomogeneity)

Fully non-perturbative!

Pair creation rate (quasiclassical approximation):

$$W_{e^-e^+} \propto \exp\left\{-\frac{\pi m^2}{eE} g(a_0)\right\}; \quad g(a_0) = \frac{2a_0}{\pi}\int_{-1}^{+1} d\zeta \sqrt{\frac{1-\zeta^2}{a_0^2+\zeta^2}}$$
$$= \frac{4a_0\sqrt{1+a_0^2}}{\pi}\left\{\mathrm{K}\left(\frac{1}{\sqrt{1+a_0^2}}\right) - \mathrm{E}\left(\frac{1}{\sqrt{1+a_0^2}}\right)\right\}$$

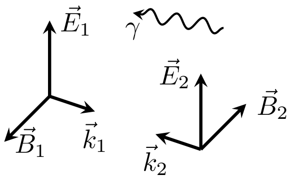
Limiting cases:

$$W_{e^-e^+} \propto \begin{cases} a_0^{2m/\omega}, & a_0 \ll 1 \quad \text{[perturbative regime];} \\ \exp\left[-\frac{\pi}{\varepsilon}\left(1 - \frac{1}{8a_0^2}\right)\right], & a_0 \gg 1 \quad \text{[non-perturbative regime].} \end{cases}$$

Drawbacks: • wrong dispersion of photons [$\vec{k} = 0$]: $\gamma \to e^- e^+$!;
• not applicable to $X-$ or γ-pulses ($\omega \ll m$)!

E. Brezin and C. Itzykson, Phys. Rev. **D2**, *1191 (1970); V. S. Popov, Sov. Phys. JETP* **34**, *709 (1972).*

Photon propagating transversely in electric field



Our case (colliding constant crossed fields+one photon from right):

- $\varepsilon = \frac{e}{m^2}\sqrt{E^2 - B^2} = \frac{2e\sqrt{E_1 E_2}}{m^2}$,
- $\eta = 0$, • $\varkappa_2 = \frac{2eE_1\omega_2}{m^3}$

A version of "imaginary time method" ($B = 0$, $\vec{k}_2 \perp \vec{E}$):

$$W_{e^-e^+} \propto \exp\left\{-2\,\mathrm{Im}\int_0^{t_*}\underbrace{[\mathcal{E}_f(t) - \mathcal{E}_i(t)]}_{\text{energy release}}\,dt\right\} \ominus \quad \left|\bullet\, \mathcal{E}_f(t_*) = \mathcal{E}_i(t_*);\right.$$

$$\bullet\, \mathcal{E}_i = \omega; \quad \bullet\, \mathcal{E}_f(t) = 2\times\sqrt{m^2 + \left(\frac{\omega}{2}\right)^2 + e^2E^2t^2}; \quad \bullet\, t_* = \frac{im}{eE}\right|$$

$$\ominus \exp\left\{-\frac{2m^2}{eE}\left[\left(1 + \left(\frac{\omega}{2m}\right)^2\right)\arctan\left(\frac{2m}{\omega}\right) - \frac{\omega}{2m}\right]\right\} \ominus \left|\bullet\, \frac{eE}{m^2} \to \varepsilon; \bullet\, \frac{\omega}{m} \to \frac{\varkappa_2}{\varepsilon}\right|$$

$$\ominus \exp\left\{-\frac{2}{\varepsilon}\left[\left(1 + \left(\frac{\varkappa_2}{2\varepsilon}\right)^2\right)\arctan\left(\frac{2\varepsilon}{\varkappa_2}\right) - \frac{\varkappa_2}{2\varepsilon}\right]\right\} \sim \begin{cases} e^{-\pi/\varepsilon}, & \varkappa_2 \ll \varepsilon \ll 1; \\ e^{-8/3\varkappa_2}, & \varepsilon \ll \varkappa_2 \ll 1. \end{cases}$$

More sophisticated derivation from $\Pi_{\mu\nu}(k;\,F)$: G.V. Dunne, H. Gies, and R. Schützhold, Phys. Rev. D ***80****, 111301(R) (2009); V.N. Baier and V.M. Katkov, Phys. Lett. A* ***374****, 2201 (2010); V.M. Katkov, JETP* ***114****, 226 (2012) [arXiv:1102.3510].*

Volkov solution (**scalar** field)

Klein-Gordon equation:

$$\{[i\partial_\mu - eA_\mu(x)]^2 - m^2\}\Psi(x) = 0$$

Plain wave field:

$$A_\mu(x) = A_\mu(\varphi),\ \varphi = k\cdot x = \omega x_-,$$
$$k\cdot A = 0$$

Volkov solution (for **scalar** field):

$$\Psi_{\vec{p}_\perp, p_-} = \frac{1}{\sqrt{2|p_-|(2\pi)^3}} \exp\left\{ -ipx - \frac{ie}{kp}\int_0^{k\cdot x} \left[pA(\varphi) - \frac{e}{2}A^2(\varphi)\right] d\varphi \right\}$$

$$p^2 = m^2, \quad p^0[\text{energy}] = \frac{1}{2}\left(p_- + \frac{m^2 + p_\perp^2}{p_-}\right)$$

Orthogonality and normalization in terms of light cone variables:

$$\int d^2x_\perp dx_- \Psi^*_{\vec{p}_\perp, p_-}\left(i\frac{\overleftrightarrow{\partial}}{\partial x_+} - eA_+\right)\Psi_{\vec{p}'_\perp, p'_-} = \mathrm{sgn}(p_-)\delta(\vec{p}_\perp - \vec{p}'_\perp)\delta(p_- - p'_-)$$

$p_- > 0$ corresponds to particles, $p_- < 0$ – to antiparticles

Exactly solvable model for counter-propagating pulses

Model external field (plane wave+"counter-propagating delta-pulse"):

$$A_\mu(x) = A_{L\mu}(x_-) + \mathcal{A}_{0\mu}\,\theta(x_+) = \begin{cases} A_{L\mu}(x_-), & x_+ < 0 \quad \text{[in-region]}, \\ A_{L\mu}(x_-) + \mathcal{A}_{0\mu}, & x_+ > 0 \quad \text{[out-region]} \end{cases}$$

In- and Out-modes:

$$\Psi^{(in)}_{\vec{p}_\perp, p_-}(x) = \Psi_{\vec{p}_\perp, p_-}(x; A_L); \quad \Psi^{(out)}_{\vec{p}_\perp, p_-}(x) = \Psi_{\vec{p}_\perp, p_-}(x; A_L + \mathcal{A}_0); \quad p_- > 0$$

Bogolubov transformation (essentially, matching condition at $x_+ = 0$):

$$\Psi_{\vec{p}_\perp, p_-}(x; A_L)|_{x_+=0} = \int d^2p'_\perp \int_{-\infty}^{+\infty} dp'_-\, \alpha_{p_-, \vec{p}_\perp; p'_- \vec{p}'_\perp}\, \Psi_{\vec{p}'_\perp, p'_-}(x; A_L + \mathcal{A}_0)|_{x_+=0}$$

Total number of created pairs:

$$N_{e^-e^+} = \int d^2p_\perp \int_0^\infty dp_- \int d^2p'_\perp \int_{-\infty}^{0} dp'_- |\alpha_{p_-, \vec{p}_\perp; p'_-, \vec{p}'_\perp}|^2$$

Particular assumptions

- Both fields are propagating along z;
- Transverse gauge is in use ($A_0 = A_3 = 0$);
- Both fields are polarized along x;
- The "soft pulse 1" is of zero frequency ($\equiv$ constant crossed field);

Vector potential:

$$\vec{A}_L(x_-) = \{-E_L x_-, 0, 0\}; \quad \vec{\mathcal{A}}_0 = \{-\mathcal{A}_0, 0, 0\}$$

Non-zero components of the EM fields:

$$E_x = E_L + \mathcal{A}_0 \delta(x_+); \quad B_y = E_L - \mathcal{A}_0 \delta(x_+)$$

- Physical interpretation of δ-function: $E_\gamma|_{x_+=0} = \mathcal{A}_0 \delta(0) = \infty \to E_\gamma = \mathcal{A}_0/\tau$; $\mathcal{A}_0$ – arbitrary; τ [pulse duration] - the smallest parameter in the problem.

Poincaré invariants:

$$E^2 - B^2 = 4E_L \mathcal{A}_0 \delta(x_+) \geq 0; \quad \vec{E} \cdot \vec{B} = 0 \implies \varepsilon|_{x_+=0} \sim \frac{2e\sqrt{E_L E_\gamma}}{m^2}, \quad \eta = 0$$

Calculations

Explicitly,

$$\Psi_{\vec{p}_\perp, p_-} = \frac{1}{\sqrt{2|p_-|(2\pi)^3}} \exp\left\{ i\vec{p}_\perp \cdot \vec{x}_\perp - i\frac{(m^2+p_\perp^2)x_-}{2p_-} - i\frac{p_- x_+}{2} + i\frac{e}{p_-}\int_0^{x_-}\left[\vec{p}_\perp \cdot \vec{A}_L(x_-) - \frac{e}{2}\vec{A}_L^{\,2}(x_-)\right] dx_- \right\}$$

Via the orthogonality& normalization condition:

$\alpha_{p_-,\vec{p}_\perp;p'_-,\vec{p}'_\perp} \propto$ [up to a constant phase factor]

$$\propto \int d^2x_\perp \int_{-\infty}^{+\infty} dx_- \left[\Psi^*_{\vec{p}'_\perp,p'_-}(x; A_L + \mathcal{A}_0) \overset{\leftrightarrow}{\frac{\partial}{\partial x_-}} \Psi_{\vec{p}_\perp,p_-}(x; A_L)\right]\Bigg|_{x_+=0}$$

$$\sim \frac{\delta(\vec{p}_\perp - \vec{p}'_\perp)}{2\sqrt{p_- q_-}} \left\{ \frac{(2p_- q_-)^{1/3}(p_- - q_-)e^2\mathcal{A}_0^2}{(eE_L)^{2/3}(p_- + q_-)^{7/3}} \mathrm{Ai}(\xi) - 2i\frac{(2p_- q_-)^{2/3} e\mathcal{A}_0}{(eE_L)^{1/3}(p_- + q_-)^{5/3}} \mathrm{Ai}'(\xi) \right\}$$

$$p'_- < 0, \quad q_- = |p'_-|, \quad \xi = \frac{(p_- + q_-)^2(m^2 + p_y^2) + e^2\mathcal{A}_0^2 p_- q_-}{(2eE_L p_- q_-)^{2/3}(p_- + q_-)^{4/3}}$$

Bogolubov coefficient $\alpha_{p_-,\vec{p}_\perp;p'_-,\vec{p}'_\perp}$ is independent on p_x, p'_x

Integration over momenta

Integration over p_x:

Classically, after pair creation $\dot{p}_x = eE_x - eB_y v_z = eE_L(1-v_z) = eE_L \dot{x}_-$. Thus, $dp_x = eE_L dx_-$ and
$\int dp_x = eE_L x_-|_{x_+=0} \sim 2eE_L \times T$ [total time of observation]

Hence:

$$\int d^2p_\perp d^2p'_\perp \delta^2(\vec{p}-\vec{p}') \ldots = \int d^2p_\perp \delta(\vec{0}) \ldots = \int \frac{dp_x dp_y S_\perp}{(2\pi)^2} \ldots = \int \frac{dp_y 2eE_L T S_\perp}{(2\pi)^2} \ldots$$

$$\frac{N_{e^-e^+}}{S_\perp \cdot T} = \frac{eE_L}{2\pi^2} \int_{-\infty}^{+\infty} dp_y \int_0^\infty dp_- \int_0^\infty dq_- \ldots$$

Change of variables: $\{p_-, q_-\} \to \{u, \lambda\}$ $[0 < u < \infty, 0 < \lambda < 1]$:

$$p_- = (1-\lambda)u^{-3/2}, \quad q_- = \lambda u^{-3/2}$$

$$\frac{\partial(p_-, q_-)}{\partial(\lambda, u)} = \frac{3}{2}u^{-4}; \quad \xi \text{ [argument of Ai's]} = \frac{m^2 + p_y^2 + e^2 \mathcal{A}_0^2 \lambda(1-\lambda)}{[2eE_L \lambda(1-\lambda)]^{2/3}} u \equiv k(\lambda) u$$

Integrals over "u": $\int_0^\infty u \,\mathrm{Ai}^2(ku)\, du = \frac{1}{6\pi\sqrt{3}k^2}, \quad \int_0^\infty \mathrm{Ai}'^2(ku)\, du = \frac{1}{3\pi\sqrt{3}k}$

The remaining integrations over p_y and λ are elementary.

Final result

$$\frac{N_{e^-e^+}}{S_\perp \cdot T} = \frac{eE_L \cdot e^2 \mathcal{A}_0^2}{8\pi^3\sqrt{3}} \int\limits_{-\infty}^{+\infty} dp_y \int\limits_0^1 d\lambda \, \frac{\lambda(1-\lambda)\left\{8(m^2+p_y^2)+e^2\mathcal{A}_0^2[1+4\lambda(1-\lambda)]\right\}}{[m^2+p_y^2+e^2\mathcal{A}_0^2\lambda(1-\lambda)]^2}$$

$$= \frac{eE_L}{4\pi^3\sqrt{3}} \int\limits_{-\infty}^{+\infty} dp_y \, \left\{1 - \frac{2(m^2+p_y^2-e^2\mathcal{A}_0^2/2)}{e\mathcal{A}_0\sqrt{m^2+p_y^2+e^2\mathcal{A}_0^2/4}} \operatorname{arth}\left(\frac{e\mathcal{A}_0/2}{\sqrt{m^2+p_y^2+e^2\mathcal{A}_0^2/4}}\right)\right\}$$

It follows that $p_x \sim eE_L T$, $p_y \sim m\sqrt{1+a_{02}^2}$, $p_- \sim \frac{[m^2(1+a_{02}^2)]^{3/2}}{eE_L}$.

Created pairs acquire “effective mass” due to acceleration/rotation inside delta-pulse!

Finally, we obtain:

$$\frac{N_{e^-e^+}}{S_\perp \cdot T} = \frac{m^3(eE_L/m^2)}{16\pi^2\sqrt{3}} F(a_{02}), \quad a_{02} = \frac{e\mathcal{A}_0}{m}$$

$$F(a_{02}) = (5a_{02} - 4/a_{02})\arctan(a_{02}/2) + 2 \approx \begin{cases} (8/3)a_{02}^2, & a_{02} \ll 1, \\ (5\pi/2)a_{02}, & a_{02} \gg 1. \end{cases}$$

No exponential suppression is present, because $\varepsilon \gg 1$ is assumed!

Interpretation of limiting case $a_{02} \ll 1$ (I)

Probability rate for scalar pair creation by photon polarized $\parallel$ to field:

$$W_{\parallel}(\omega) = -\frac{e^2 m^2}{2\omega\sqrt{\varkappa}} \int\limits_{(4/\varkappa)^{2/3}}^{\infty} d\zeta \, \frac{(\kappa\zeta^{3/2}+2)\,\mathrm{Ai}'(\zeta)}{\zeta^{11/4}\sqrt{\varkappa\zeta^{3/2}-4}}, \quad \varkappa = \frac{eE_L k_-}{m^3} = \frac{2eE_L\omega}{m^3}$$

A.I. Nikishov and V.I. Ritus, Sov. Phys. JETP ***25****, 1135 (1967).*

Photon spectrum in a δ-pulse:

$$\vec{E}_\gamma = \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}} \frac{(-i\omega_k)\sqrt{4\pi}}{\sqrt{2\omega_k}} \left\{ \vec{e}_{\vec{k}\lambda} c_{\vec{k}\lambda} e^{-ikx} - \vec{e}^{\,*}_{\vec{k}\lambda} c^\dagger_{\vec{k}\lambda} e^{+ikx} \right\} = \vec{\mathcal{A}}_0 \delta(x_+)$$

$$c_{\vec{k}\lambda} = i\delta_{\lambda,\parallel} \frac{\delta(\vec{k}_\perp)\theta(-k_z)}{\sqrt{\omega_k}},$$

$$dN_\gamma(\omega) = |c_{\vec{k},\parallel}|^2\, d^3k = \frac{S_\perp \mathcal{A}_0^2 \delta(\vec{k}_\perp)\theta(-k_z)}{(2\pi)^2\omega_k}\, d^2k_\perp dk_z = \frac{S_\perp \mathcal{A}_0^2\, d\omega}{(2\pi)^2\omega}$$

Number of created pairs (1st order of PT with respect to γ-pulse):

$$\frac{N_{e^-e^+}}{S_\perp \cdot T} = \frac{1}{S_\perp} \int\limits_0^\infty W_\parallel(\omega) \frac{dN_\gamma(\omega)}{d\omega}\, d\omega$$

Interpretation of limiting case $a_{02} \ll 1$ (II)

Calculation:

$$\frac{N_{e-e+}}{S_\perp \cdot T} = \frac{1}{S_\perp}\int_0^\infty W_\parallel(\omega)\frac{dN_\gamma(\omega)}{d\omega}\,d\omega$$

$$= -\frac{e^2m^2\mathcal{A}_0^2}{8\pi^2}\int_0^\infty \frac{d\omega}{\omega^2\sqrt{\varkappa}}\int_{(4/\varkappa)^{2/3}}^\infty d\zeta\,\frac{(\varkappa\zeta^{3/2}+2)\,\mathrm{Ai}'(\zeta)}{\zeta^{11/4}\sqrt{\varkappa\zeta^{3/2}-4}} = \left|\bullet\ \omega\to\varkappa=\frac{2eE_L\omega}{m^3}\right|$$

$$\ominus -\frac{(eE_L)(e\mathcal{A}_0)^2}{(2\pi)^2m}\int_0^\infty\frac{d\varkappa}{\varkappa^{5/2}}\int_{(4/\varkappa)^{2/3}}^\infty d\zeta\,\frac{(\varkappa\zeta^{3/2}+2)\,\mathrm{Ai}'(\zeta)}{\zeta^{11/4}\sqrt{\varkappa\zeta^{3/2}-4}} = \left|\bullet\ \text{swapping order of integrations}\right|$$

$$\ominus -\frac{(eE_L)(e\mathcal{A}_0)^2}{(2\pi)^2m}\int_0^\infty\frac{d\zeta}{\zeta^{11/4}}\mathrm{Ai}'(\zeta)\int_{4/\zeta^{3/2}}^\infty\frac{d\varkappa}{\varkappa^{5/2}}\frac{(\varkappa\zeta^{3/2}+2)}{\sqrt{\varkappa\zeta^{3/2}-4}} =$$

$$= \left|\bullet\int_{4/\zeta^{3/2}}^\infty\frac{d\varkappa}{\varkappa^{5/2}}\frac{(\varkappa\zeta^{3/2}+2)}{\sqrt{\varkappa\zeta^{3/2}-4}} = \frac{2}{3}\zeta^{9/4},\ \bullet\int_0^\infty\frac{d\zeta}{\sqrt{\zeta}}\mathrm{Ai}'(\zeta) = -\frac{1}{\sqrt{3}}\right| \ominus \frac{(eE_L)(e\mathcal{A}_0)^2}{6\pi^2\sqrt{3}\,m}$$

Exactly coincides with our limiting case $a_{02} \ll 1$! $N_{e_-e_+} \propto a_{02}^2 \propto \bar{N}_\gamma$

Interpretation of limiting case $a_{02} \gg 1$

$a_{02} = eE_\gamma \tau_2/m \gg 1$ – compare to "locally constant field approximation":

$$N_{e^-e^+} = \int dt\, dV \frac{m^4 \varepsilon(z,t)\eta(z,t)}{8\pi^2} \sinh^{-1}\left(\frac{\pi\eta(z,t)}{\varepsilon(z,t)}\right) \exp\left(-\frac{\pi}{\varepsilon(z,t)}\right)$$

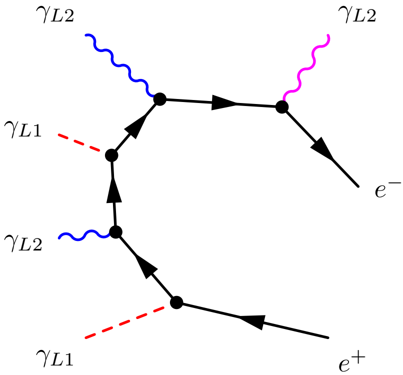
$$|\bullet \eta = 0| \ominus \int dt\, dV \frac{m^4 \varepsilon^2(z,t)}{8\pi^3} \exp\left(-\frac{\pi}{\varepsilon(z,t)}\right) \quad \left|\bullet \varepsilon = \frac{2e\sqrt{E_L \mathcal{A}_0 \delta(x_+)}}{m^2}\right|$$

$$\ominus \int dt\, dV \frac{e^2 E_L \mathcal{A}_0}{2\pi^3} \delta(x_+) \cancelto{1}{\exp\left(-\frac{\pi}{\varepsilon(x_+)}\right)} \sim (2\pi^3)^{-1} (eE_L)(e\mathcal{A}_0) S_\perp T.$$

Functional dependence is the same as in our limiting case $a_{02} \gg 1$!
Difference: $(2\pi^3)^{-1} \approx 1.61 \times 10^{-2}$ instead of $\frac{5}{32\pi\sqrt{3}} \approx 2.87 \times 10^{-2}$.

$N_{e^-e^+} \propto a_{02} \propto \sqrt{\bar{N}_\gamma} \sim \Delta N_\gamma$ (assuming Poisson distribution) –???

$N_{e^-e^+} \propto \Delta N_\gamma$ for $\bar{N}_\gamma \gg 1$ – explanation



- Non-perturbative regime
- Photons are in fact both **emitted** and **absorbed**
- **Harder photons** are mostly **absorbed**; **softer photons** are mostly **emitted**
- The **net number of absorbed photons** is related to ΔN_γ

Summary

(Seemingly) first *exactly solvable* model for pair creation by colliding pulses. Pros and Cons:

- Utilization of δ-pulse in a spirit of TM power-duration conjecture
- However, required scaling with τ_γ [pulse duration] ($E_\gamma \propto \tau_\gamma^{-1}$; $\mathcal{E}_\gamma \propto \tau_\gamma^{-1}$) is more singular than for TM ($E_\gamma \propto \tau_\gamma^{-1/2}$; $\mathcal{E}_\gamma = \mathrm{const}$)
- Truly non-perturbative behavior:
  - footprints of effective mass in momentum distribution
  - matching of complimentary asymptotic expansions [*locally constant field approximation* vs *1st order of perturbation theory for δ-pulse*] at $a_{02} \sim 1$
- Unfortunately, exponential suppression (crucial for realistic [experimental] setups) is missing in this regime - no light is shed on correction of tunneling probabilities by field variation
- Possible application: understanding of correct description for back reaction [field shielding and dumping by creating plasma of pairs]

Outlook

Possible further generalizations:

- **Creation of fermion pairs**
- **Arbitrary** $A_\mu(x_-)$
- Arbitrarily rotated polarization of δ-pulse
- Oblique incidence of δ-pulse
- Train of δ-pulses
- **Back reaction-?**
- ...