Testing Strong-Field CED and QED with Intense Laser Fields

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Abstract: The feasibility of testing classical electrodynamics (CED) and quantum electrodynamics (QED) under the extreme conditions produced by already- and soon-available intense laser fields is investigated. Different processes are studied, envisaging for the first time the possibility of measuring experimentally effects due to radiation reaction in CED and to the polarization of the vacuum in QED at laser intensities larger or of the order of \( 10^{22} \text{ W/cm}^2 \).

1. Introduction

Classical electrodynamics (CED) and quantum electrodynamics (QED) are well established theories and have been successfully tested experimentally in different regimes. However, there are still areas of CED and QED that deserve theoretical and experimental investigation. In view of the increasingly stronger available laser fields it is becoming feasible to employ them to test CED and QED under the extreme conditions supplied by ultra-intense fields [1]. Below we first consider the long-standing problem of radiation reaction in CED and a possible way of testing the underlying equations experimentally. Then, we put forward a scheme suitable, in principle, for measuring for the first time the vacuum-mediated quantum interaction between real photons in vacuum.

2. Radiation reaction effects in CED

A fundamental problem in electrodynamics is the so-called “radiation reaction” problem: classically, when a charged particle (an electron, for instance) is accelerated by an external field, it emits radiation and this emission changes the trajectory of the electron. The Lorentz-Abraham-Dirac (LAD) equation consistently describes the motion of an electron in an external electromagnetic field by taking into account radiation reaction [3]. The LAD equation is one of the most controversial equations in physics due to its allowance of runaway solutions and pre-acceleration effects. However, it can be shown that in the realm of CED, this equation can be consistently reduced to the so-called Landau-Lifshitz (LL) equation that does not show the mentioned problems. If \( u^{\mu} \) is the four velocity of the electron (with charge \(-e<0\) and mass \( m \)) and \( F^{\mu\nu}(x) \) is the external electromagnetic field depending on the coordinates \( x=x^{\mu} \), the LL equation reads (the units \( h/2\pi=c=1 \) are used throughout) [2]

\[
m\frac{du^{\mu}}{d\tau} = -eF^{\mu\nu}u_{\nu} - \frac{2}{3} \alpha \left[ \frac{e}{m} \partial_{\alpha}(F^{\mu\nu})u^{\alpha}u_{\nu} + \frac{e^2}{m^2} F^{\mu\alpha}F_{\alpha\nu}u^{\alpha}u_{\nu} - \frac{e^2}{m^2} (F^{\mu\alpha}u_{\alpha})(F_{\alpha\lambda}u^{\lambda})u^{\mu} \right],
\]

where \( \tau \) is the proper time of the electron and \( \alpha = e^2/4\pi \) is the fine-structure constant. The above LL equation can be solved exactly and analytically if the external field is a plane wave [3]. In this case the solution shows that the radiation reaction effects scale with the parameter \( R = (2/3)\alpha \xi \) where \( \xi = eE/m\omega \) is the adimensional vector potential of the plane wave, with \( E \) and \( \omega \) being the electric field amplitude and the central frequency of the wave and where \( \chi \) is the electric field amplitude of the wave in the rest frame of the electron (with respect to its initial velocity) in units of the critical field \( E_{\text{cr}} = m^2/e = 1.3*10^{16} \text{ V/cm} \) [1]. It is interesting that it is possible to achieve \( R \approx 1 \) also when quantum effects are negligible (i.e., if \( \chi \ll 1 \)). This is the so-called “radiation-dominated” regime where the effects of radiation reaction are rather large. However, in order to realize experimentally the radiation-dominated regime, either very intense lasers or very high-energy electron beams or both are required. In [4] we have found an alternative physical regime where, although \( R \ll 1 \), the effects of radiation reaction in the energy spectra emitted by laser-driven electrons are manifest. In this regime the average momentum transferred by the laser field in one period to the initially counter-propagating electron almost compensates for the initial momentum of the electron along the laser propagation direction. This parameter
regime appears to be very sensitive to radiation reaction. We have shown that the angular distribution of the radiation emitted by one electron is strongly modified if the radiation reaction terms in the LL equation are taken into account. The induced modifications are shown to be in principle measurable at laser intensities of the order of $10^{22}$ W/cm$^2$ and electron energies of the order of tenths of MeVs. This could represent the first experimental test of the LL equation.

3. Vacuum polarization effects in QED
In CED electromagnetic waves do not interact with each other in vacuum: the superposition principle holds. Instead, QED predicts that the structure of the vacuum is more complicated and “virtual” electron-positron pairs continuously create and annihilate in vacuum. These evanescent pairs may allow, in principle, the interaction between photons in the vacuum (photon-photon scattering) [5]. Under certain approximations [5] this interaction can be described for a generic electromagnetic field $(\mathbf{E}, \mathbf{B})$ by the following effective Lagrangian density [5]

$$L = \frac{1}{2} \left( E^2 - B^2 \right) + \frac{2\alpha^2}{45m^4} \left[ (E^2 - B^2)^2 + 7(E \cdot B)^2 \right].$$

In [6] we have envisaged the possibility of testing these quantum corrections to the classical Lagrangian density $(E^2-B^2)/2$ in a “matterless” double-slit scenario. Two strong parallel laser beams collide head-on with a probe electromagnetic field. Each photon in the probe may interact through the “polarized” quantum vacuum with the photons of the other two fields. Analogous to “ordinary” double-slit set-ups involving matter, the vacuum-scattered probe photons produce a diffraction pattern which is the envisaged observable to measure the quantum interaction between the probe and strong field photons. In this case we have shown that the diffraction pattern becomes visible in a few operating hours if the strong fields have an intensity as high as those envisaged at the Extreme Light Infrastructure (ELI) and at the High Power Laser Energy Research (HiPER) facilities, i.e. of the order of $10^{24}$-$10^{25}$ W/cm$^2$ [6].

4. Conclusion
In conclusion, the advent of ultra-intense laser sources may allow in the near future the experimental investigations of a number of fascinating problems in classical and quantum electrodynamics. In particular, we have shown that the Landau-Lifshitz equation can be in principle tested experimentally and that vacuum-induced real photon-photon scattering could be measured for the first time.