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Testing strong-field CED and QED with intense laser fields

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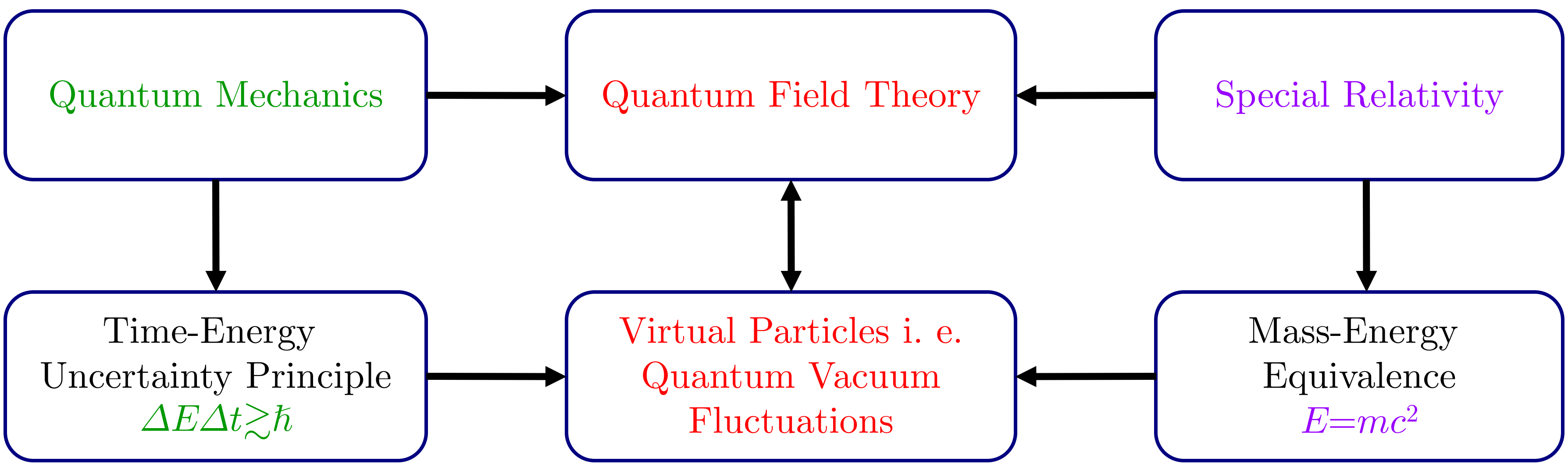
OUTLINE

- Classical vs quantum vacuum:
  - A matterless double-slit
- Radiation reaction in classical electrodynamics:
  - Strong signature of radiation reaction below the radiation dominated regime
- Conclusions

Classical vs. quantum vacuum

- In quantum field theory the vacuum state is the state in which no real particles are present (electrons, positrons, photons etc...)
  - Virtual particles are present
  - They live for a very short time and cover a very short distance ($\tau = \hbar/mc^2$ and $\lambda_c = \hbar/mc$, respectively). For electrons and positrons: $\lambda_c \sim 10^{-11}$ cm and $\tau \sim 10^{-21}$ s.



- Can we somehow detect the presence of the virtual particles?

Critical fields

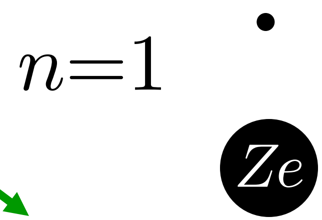
Since virtual particles live for a very short time, then very strong fields are needed to make apparent the effects of their presence

A strength scale is given by the critical fields

$$E_{cr} = \frac{m^2c^3}{\hbar e} = 1.3 \times 10^{16}\ \mathrm{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar e} = 4.4 \times 10^{13}\ \mathrm{G}$$

$$I_{cr} = \frac{cE_{cr}^2}{8\pi} = 2.3 \times 10^{29}\ \mathrm{W/cm^2}$$



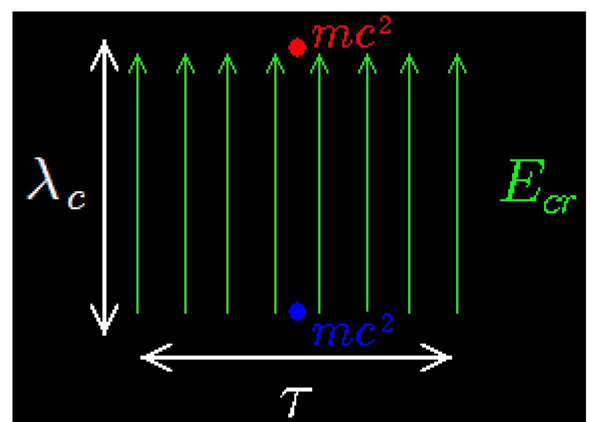
$$d = \frac{a_0}{Z} = \frac{\lambda_c}{Z\alpha}$$

$$E = \frac{Ze}{d^2} = (Z\alpha)^3 E_{cr}$$

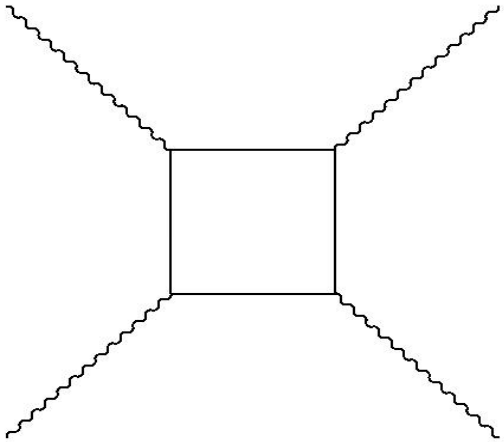
Physical meaning of the critical fields:

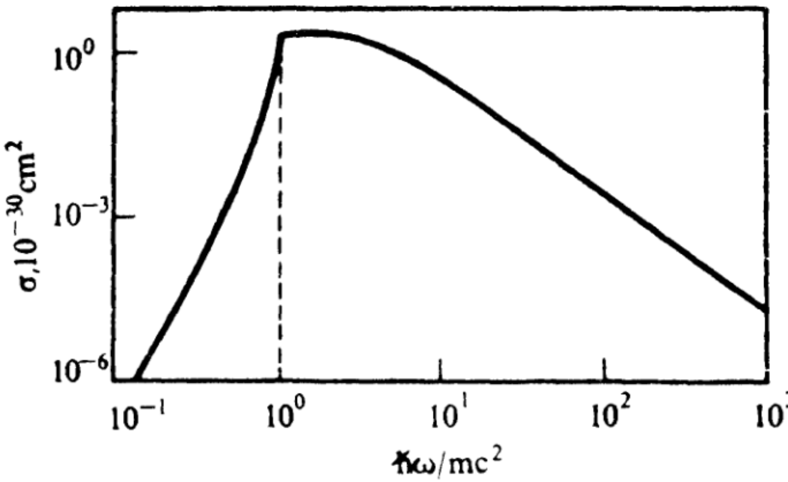
$$\frac{\hbar}{mc} \times eE_{cr} \sim mc^2$$

$$\frac{e\hbar}{mc} \times B_{cr} \sim mc^2$$



Elastic photon-photon scattering





- The total cross section of this process is given by (in the center-of-mass reference)

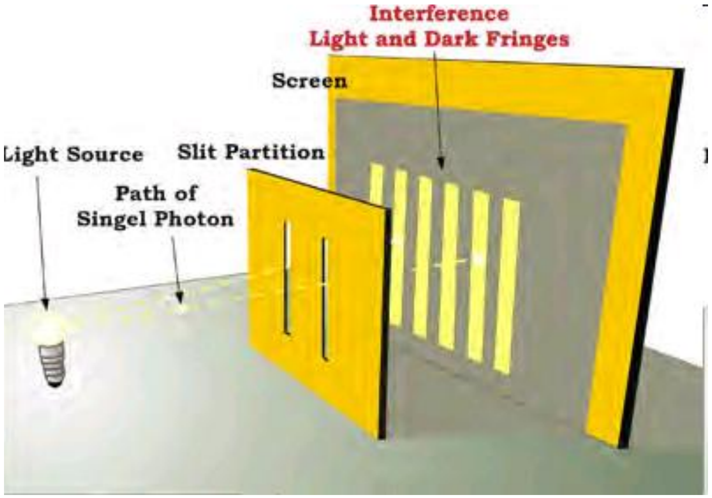
$$\sigma = \begin{cases} 3\times 10^{-2}\alpha^4\lambda_c^2\left(\frac{\hbar\omega}{mc^2}\right)^6 & \text{if } \hbar\omega \ll mc^2 \\ 4.7\alpha^4\left(\frac{c}{\omega}\right)^2 & \text{if } \hbar\omega \gg mc^2 \end{cases}$$

- The maximum is "only" 10^{-5} times the total cross section of Thomson scattering but it is in the MeV range
- Steep dependence on $(\hbar\omega/mc^2)^6$ at small energies
- Background

Can the large number of photons in strong optical laser beams compensate for the $(\hbar\omega/mc^2)^6$-suppression?

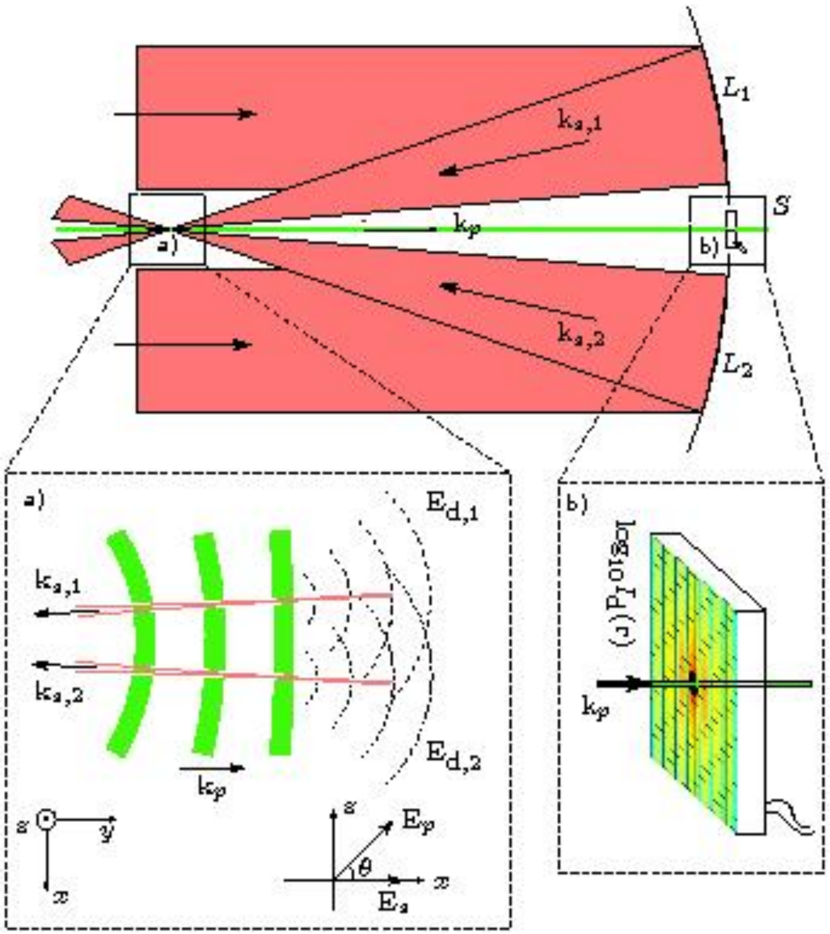
A matterless double-slit

- The double slit experiment has played a fundamental role in our understanding of quantum mechanics, in particular the so-called wave-particle duality of particles

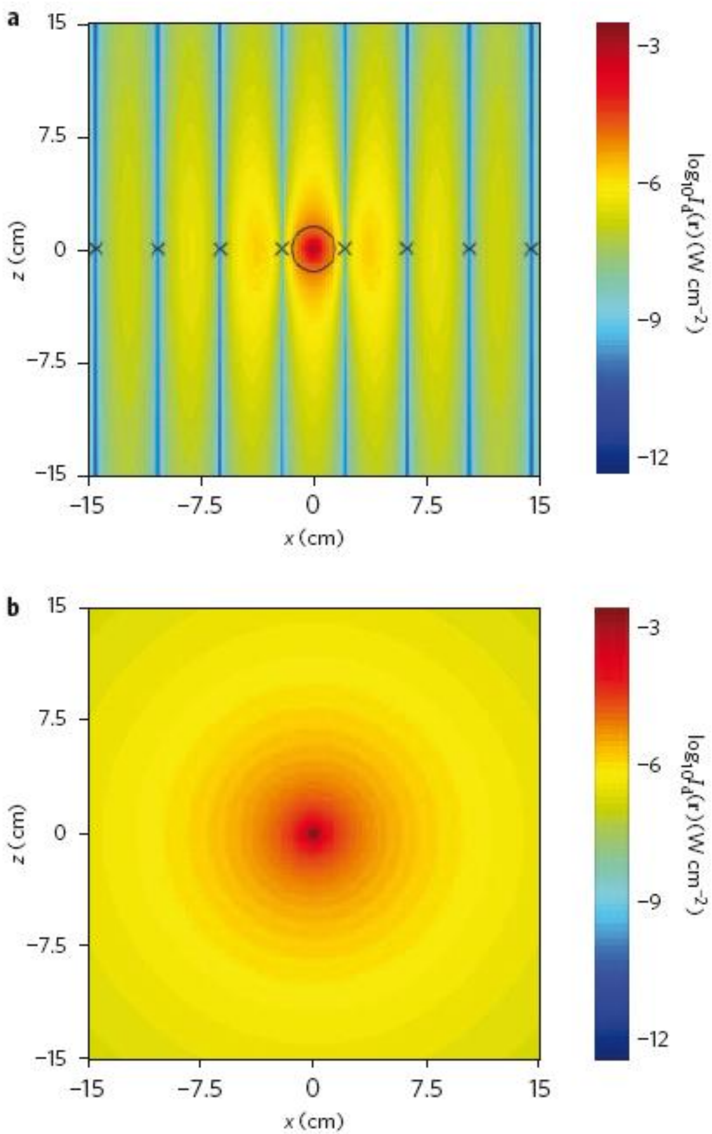


- All double-slit schemes proposed so far have always involved matter (either the particles employed like electrons, neutrons and so on or the wall where the double slit is)

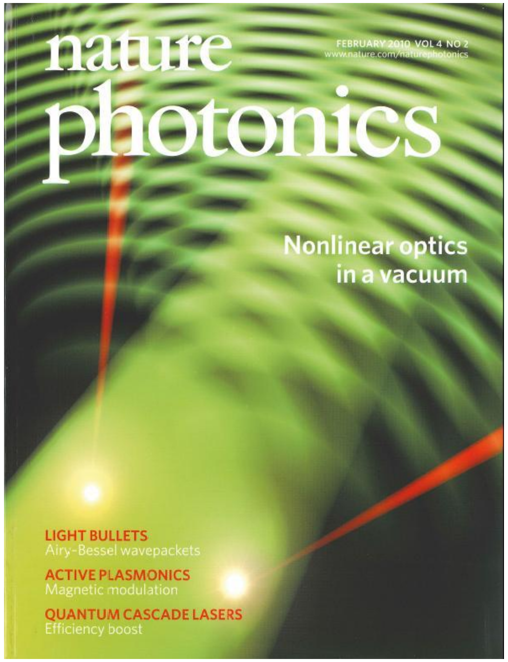
- In QED the virtual particles present in the vacuum can mediate an interaction between electromagnetic fields in vacuum (in CED electromagnetic fields do not interact in the vacuum)
- Under certain approximations one can describe the quantum vacuum in the presence of a strong electromagnetic field (like that of a focused laser field) as a piece of dielectric with refractive index different from unity



Results



- With ELI parameters a few photons per shot are scattered
- The interference spectrum is built in about four hours of operation (theoretical visibility about 48 %)



B. King, A. Di Piazza and C. H. Keitel, Nature Photon. **4**, 92 (2010)

Radiation reaction in classical electrodynamics

What is the equation of motion of an electron in an external, given electromagnetic field $F^{\mu\nu}(x)$?

- The Lorentz equation

$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu$$

  does not take into account that while being accelerated the electron generates an electromagnetic radiation field and it looses energy and momentum

- One has to solve the coupled Lorentz and Maxwell equations

$$m_0\frac{du^\mu}{ds} = -eF_T^{\mu\nu}u_\nu$$

$$\partial_\mu F_T^{\mu\nu} = -e\int ds\delta(x - x(s))u^\nu$$

  where now ${F_T}^{\mu\nu}(x)$ is the total electromagnetic field (external plus the one generated by the electron)

- One can solve Maxwell equations exactly with the Green function method

$$m_0 \frac{du^\mu}{ds} = -eF_T^{\mu\nu} u_\nu$$

$$\partial_\mu F_T^{\mu\nu} = -e \int ds \delta(x - x(s)) u^\nu \quad \longrightarrow \quad F_T^{\mu\nu}(x) = F^{\mu\nu}(x) + F_S^{\mu\nu}(x)$$

and re-substitute the total field into the Lorentz equation:

$$(m_0 + \delta m) \frac{du^\mu}{ds} = -eF^{\mu\nu} u_\nu + \frac{2}{3}\alpha \left(\frac{d^2 u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds} u^\mu \right)$$

- By renormalizing the electron mass (note that δm is infinite!) one obtains the Lorentz-Abraham-Dirac equation

$$m \frac{du^\mu}{ds} = -eF^{\mu\nu} u_\nu + \frac{2}{3}\alpha \left(\frac{d^2 u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds} u^\mu \right)$$

- It is known that the Lorentz-Abraham-Dirac equation

$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu + \frac{2}{3}\alpha\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

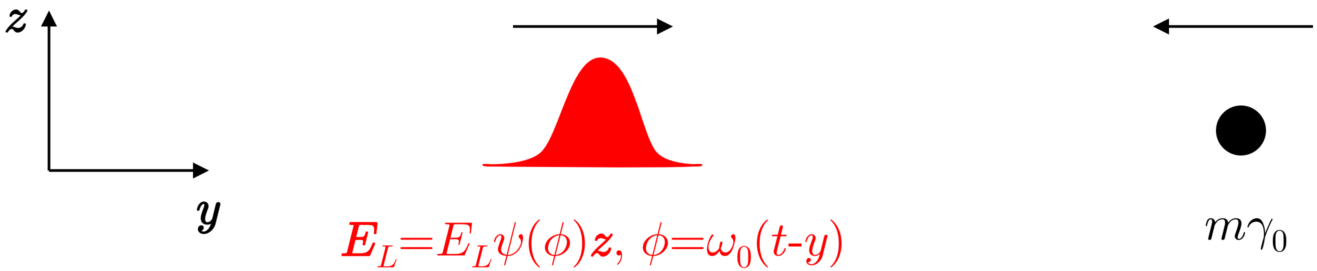
is plagued by serious problems due to the presence of the derivative of the acceleration: runaway solutions, preacceleration

- If in the instantaneous rest frame of the electron the field's amplitude $F^* \ll F_{cr}/\alpha \approx 137 F_{cr}$ and the field's wavelength $\lambda^* \gg \alpha\lambda_c \approx \lambda_c/137$, one can perform a "reduction of order" (Landau and Lifshitz 1947) and obtains

$$m\frac{du^\mu}{ds} = -eF^{\mu\nu}u_\nu - \frac{2}{3}\alpha\left[\frac{e}{m}(\partial_\alpha F^{\mu\nu})u^\alpha u_\nu + \frac{e^2}{m^2}F^{\mu\nu}F_{\alpha\nu}u^\alpha - \frac{e^2}{m^2}(F^{\alpha\nu}u_\nu)(F_{\alpha\lambda}u^\lambda)u^\mu\right]$$

- This is always true in the realm of classical electrodynamics because in order to neglect quantum effects it must be: $F^* \ll F_{cr}$ and $\lambda^* \gg \lambda_c$

- If $F^{\mu\nu}(x)$ is a plane wave the Landau-Lifshitz equation can be solved exactly (ADP, Lett. Math. Phys. **83**, 305 (2008))



$$u^\mu(\phi) = \frac{1}{h(\phi)} \begin{pmatrix} \gamma_0 + \frac{1}{2\gamma_0(1+\beta_0)}[h^2(\phi) - 1 + \xi^2\mathcal{I}^2(\phi)] \\ 0 \\ -\beta_0\gamma_0 + \frac{1}{2\gamma_0(1+\beta_0)}[h^2(\phi) - 1 + \xi^2\mathcal{I}^2(\phi)] \\ -\xi\mathcal{I}(\phi) \end{pmatrix} \qquad \begin{aligned} h(\phi) &= 1 + R\int_{\phi_0}^{\phi} d\zeta \psi^2(\zeta), \\ \mathcal{I}(\phi) &= \int_{\phi_0}^{\phi} d\zeta \left[h(\zeta)\psi(\zeta) + \frac{R}{\xi^2}\frac{d\psi(\zeta)}{d\zeta}\right] \end{aligned}$$

- Radiation reaction effects scale with the parameter R

$$R = \frac{2}{3}\alpha\frac{\omega_0}{m}\gamma_0(1+\beta_0)\xi^2$$

and the condition $R \approx 1$ means that the energy emitted by the electron in one laser period is of the order of the initial energy (radiation dominated regime)

- The condition $R \approx 1$ requires either GeV electrons or multipetawatt lasers
- However, from the solution

$$u^\mu(\phi) = \frac{1}{h(\phi)} \begin{pmatrix} \gamma_0 + \frac{1}{2\gamma_0(1+\beta_0)}[h^2(\phi) - 1 + \xi^2 \mathcal{I}^2(\phi)] \\ 0 \\ -\beta_0\gamma_0 + \frac{1}{2\gamma_0(1+\beta_0)}[h^2(\phi) - 1 + \xi^2 \mathcal{I}^2(\phi)] \\ -\xi \mathcal{I}(\phi) \end{pmatrix}$$

one can see that if $2\gamma_0 \approx \xi$ then the initial longitudinal momentum of the electron is almost compensated by the laser field and this regime is very sensitive to radiation reaction

- If the much less restrictive condition

$$R \gtrsim \frac{4\gamma_0^2 - \xi^2}{2\xi^2} > 0$$

is fulfilled (longitudinal momentum lost in one laser period comparable with the momentum in the laser field) the electron is reflected by the laser field only if radiation reaction is taken into account

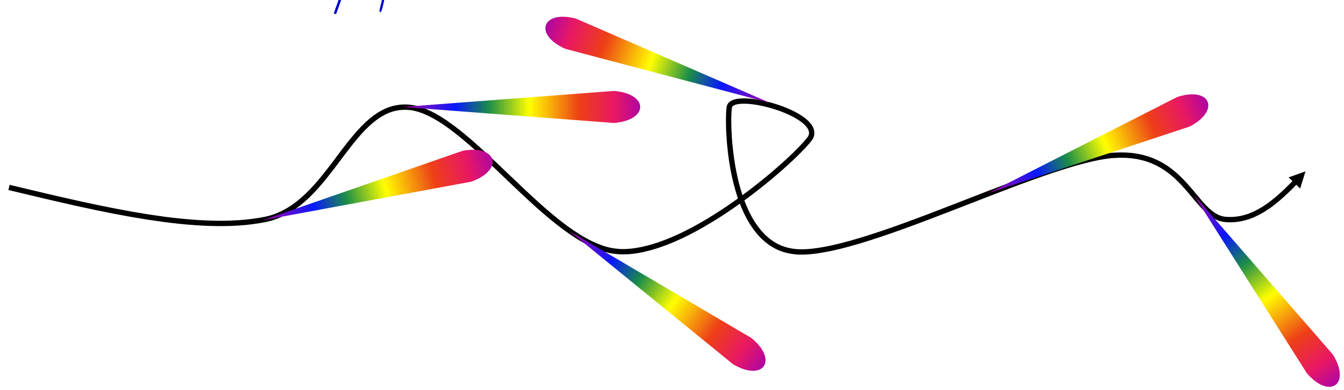
ADP, K. Z. Hatsagortsyan and C. H. Keitel, PRL **102**, 254802 (2009)

- Radiation field emitted by an electron (Landau and Lifshitz 1947):

$$\mathbf{E}_{\mathrm{rad}}(\mathbf{r},t) = -e\,\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})]\times\dot{\boldsymbol{\beta}}}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3\mathcal{R}}\bigg|_{\mathrm{ret}}$$

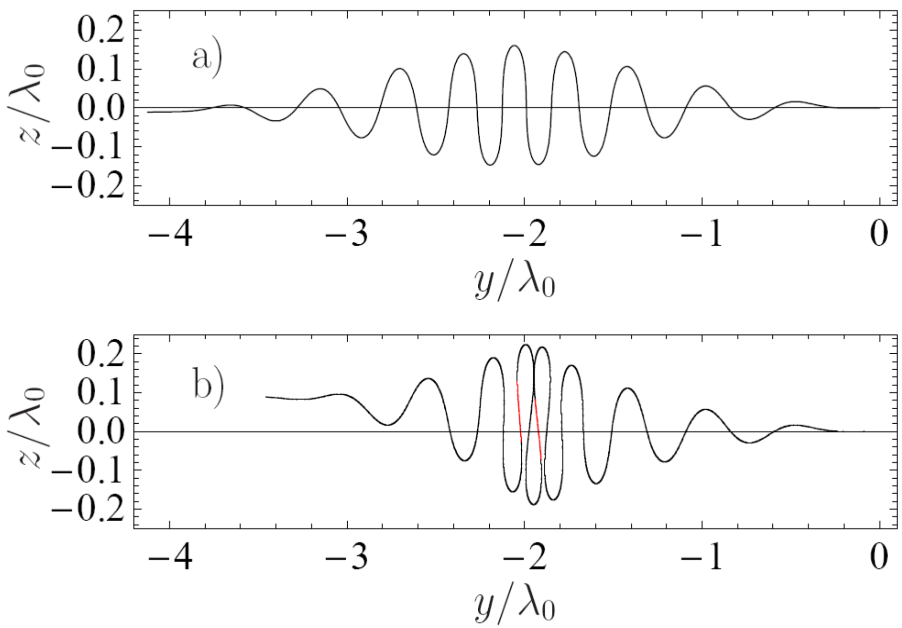
$$\mathbf{B}_{\mathrm{rad}}(\mathbf{r},t) = \mathbf{n}\times\mathbf{E}_{\mathrm{rad}}(\mathbf{r},t)$$

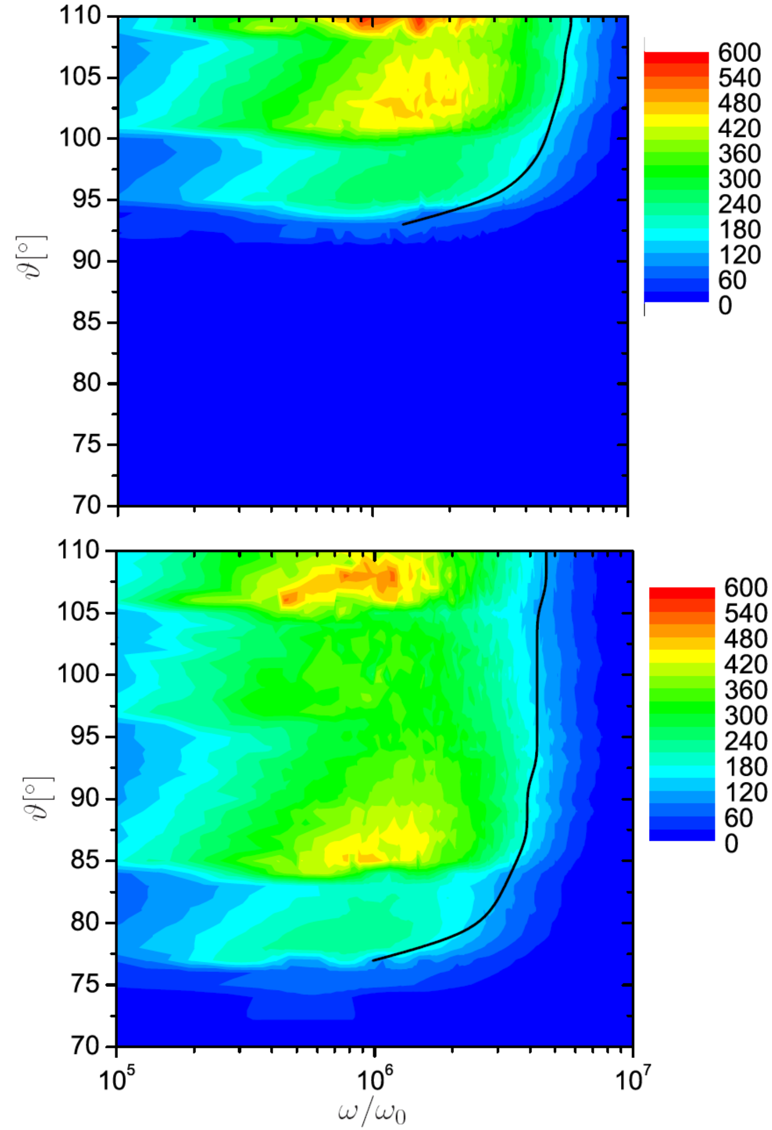
- Main features of the radiation emitted by an ultra-relativistic electron:
  - radiation emitted at each instant mainly along the electron's velocity within a cone with an aperture of the order of $1/\gamma$



  - radiation emitted at each instant with frequencies up to ω_c=$3\gamma^3/\rho$, with ρ the curvature radius

- Numerical parameters: electron energy 40 MeV ($2\gamma_0$=156), laser wavelength 0.8 μm, laser intensity 5×10^{22} W/cm^2 (ξ=150), pulse duration 30 fs, focused to 2.5 μm (note that R=0.05!)
- Electron trajectories and emission spectra without and with radiation reaction





- The red parts of the trajectory are those where the longitudinal velocity of the electron is positive
- The black lines indicate the cut-off position from the formula $\omega_c=3\omega_0\gamma^3$

ADP, K. Z. Hatsagortsyan and C. H. Keitel, PRL **102**, 254802 (2009)

Experimental observability

- With the above parameters we expect about one photon per electron to be emitted in one pulse at ϑ=80°
- Laser plasma accelerators provides electron beams with 10^9 electrons in a volume of the order of 1000 μm^3
- By accounting for the efficiency of Germanium detectors of the order of 10^{-3} one obtains about 10^4 photons per shot
- Collective effects: coherence effects at so high frequency are negligible (the typical distance between two electrons in the beam is about 10 nm), the Coulomb interaction between the electrons is negligible
- Multiparticle effects: we have changed the initial conditions of the electron (position and velocity) and have shown the effect to be rather insensitive to them
- More realistic simulations: PIC codes

CONCLUSIONS

- Classical and quantum electrodynamics are well established theories but there are still areas to be investigated theoretically and experimentally
- Intense laser fields can be employed
  - to study the properties of the quantum vacuum
  - to test the validity of the Landau-Lifshitz equation